Research and Communications in Mathematics and Mathematical Sciences
Vol. 8, Issue 2, 2017, Pages 133-142
ISSN 2319-6939
Published Online on September 14, 2017

http://jyotiacademicpress.org

TWO-SOLITARY WAVE SOLUTION OF A $(2+1)$-DIMENSIONAL GENERALIZED KP EQUATION WITH GENERAL VARIABLE COEFFICIENTS

XIANG-ZHENG LI[1], **JIN-LIANG ZHANG**[1], **and MING-LIANG WANG** [2]

[1] School of Mathematics and Statistics
Henan University of Science and Technology
Luoyang, 471023
P. R. China
e-mail: xiangzheng_li@qq.com

[2] School of Mathematics and Statistics
Lanzhou University
Lanzhou, 730000
P. R. China

Abstract

A generalized KP equation with general variable coefficients (gvcKP) has been reduced to the variable coefficients KdV equation (vcKdV) by a transformation of variables. Since the single solitary wave solution and 2-solitary wave solution of the vcKdV have been known already, substituting the solutions of the vcKdV

2010 Mathematics Subject Classification: 34A34, 34A05, 74J35.

Keywords and phrases: 2-solitary wave solution, generalized KP equation with general variable coefficients, vcKP equation, cylindrical KP equation, reduction of dimensionality, transformation of variables.

Project supported by the National Natural Science Foundation of China (Grant Nos. 11301153, 11601225) and The Doctoral Foundation of Henan University of Science and Technology (Grant No. 09001562) and The Science and Technology Innovation Platform of Henan University of Science and Technology (Grant No. 2015XPT001).

Received July 1, 2017; Revised August 16, 2017

into the corresponding transformation of variables, the single and 2-solitary wave solutions of the gvcKP can be obtained successfully.

1. Introduction

It is known that the transverse perturbations always exist in the higher-dimensional system. Anisotropy is introduced into the system and the wave structure and stability are modified by the transverse perturbation. Recent theoretical studies for ion-acoustic/dust-acoustic waves show that the properties of solitary waves in bounded non-planar cylindrical/spherical geometry differ from that in unbounded planar geometry. A dissipative cylindrical/spherical KdV is obtained by using the standard reductive perturbation method [1]. The cylindrical KP equation (CKP) has been introduced by Johnson to describe surface wave in a shallow incompressible fluid [2-3]. Since the variable-coefficient equations can model more complex physical phenomena than their constant-coefficient counterparts, variable coefficient KP equations with various form have been investigated by some authors [4-7]. With the aid of Mathematica, the idea of improved homogeneous balance method, Yan investigated a $(2+1)$-dimensional variable coefficients KP equation, found its Bäcklund transformation via which some exact solutions are obtained, and found its non-local symmetry [4]. Ma et al. found non-auto-Bäcklund transformation for $(2+1)$-dimensional generalized KP equation with variable coefficients, and obtained its symmetry transformation and exact solutions [5].

In recent years, many famous methods which can be used to find exact solutions of nonlinear partial differential equations have been proposed ([8] and references there in).

We consider the generalized KP equation with general variable coefficients (gvcKP) in the form [7]

$$[u_t + g(t)(6uu_x + u_{xxx}) + 6f(t)g(t)u]_x + h(t)u_{yy} = f'(t) + 12g(t)f^2(t), \quad (1)$$

where $g(t)$, $f(t)$, and $h(t)$ are all functions of variable t only. The gvcKP is a better integrable water wave model.

The gvcKP Equation (1) includes the following important cases:

(a) If $g(t) = 1$, $f(t) = \frac{1}{12t}$ and $h(t) = \frac{3\alpha^2}{t^2}$, Equation (1) becomes the CKP that has been investigated in [9-11].

$$\left(u_t + 6uu_x + u_{xxx} + \frac{1}{2t}u\right)_x + \frac{3\alpha^2}{t^2}u_{yy} = 0, \tag{2}$$

where α is a constant. CKP Equation (2) has been investigated to obtain decay mode solutions by means of Hirota method [9] and simplified homogeneous balance method [10] respectively, and to obtain the single and 2-solitary wave solution by means of reduction of dimensionality [11].

(b) If $f(t) = 0$, Equation (1) becomes the variable coefficient KP equation (vcKP)

$$[u_t + g(t)(6uu_x + u_{xxx})]_x + h(t)u_{yy} = 0. \tag{3}$$

In particular, if $g(t) = 1$, $h(t) = \sigma = const$, Equation (3) becomes the classical KP equation. The KP equation is also derived using reductive perturbation method in superthermal dusty plasma and the steady state solution has been given [11-13].

The present paper is a direct continuation of our previous work [11, 14]. We aim to find solitary wave solutions of gvcKP (1). The paper is organized as follows: In Section 2, the gvcKP (1) is reduced to the variable coefficient KdV equation by a transformation of variables. In Section 3, the single solitary wave solution and 2-solitary wave solution of gvcKP (1) can be obtained in terms of the corresponding transformation of variables, since the solutions of the vcKdV equation have been known already. In Section 4, some conclusions are made.

2. Reduction of gvcKP

In Equation (1), we assume that

$$u(x,\ y,\ t)=w(\xi,\ t),\ \xi=x+q(y,\ t), \tag{4}$$

where $q=q(y,\ t)$ is a function to be determined later. Substituting (4) into (1), yields a equation as follows:

$$\frac{\partial}{\partial\xi}[w_t+g(t)(6ww_\xi+w_{\xi\xi\xi})]+[6f(t)g(t)+h(t)q_{yy}]w_\xi+\left[q_t+h(t)q_y^2\right]w_{\xi\xi}$$

$$=f'(t)+12g(t)f^2(t). \tag{5}$$

Setting the coefficients of w_ξ and $w_{\xi\xi}$ to zero, yields

$$6f(t)g(t)+h(t)q_{yy}=0,\ q_t+h(t)q_y^2=0. \tag{6}$$

The system (6) admits the following solution:

$$q(y,\ t)=-\frac{3f(t)g(t)}{h(t)}y^2+c_1(t)y+c_2(t), \tag{7}$$

where $c_1(t)=k_1e^{12\int^t f(\tau)g(\tau)d\tau}$, $c_2(t)=-k_1^2\int^t h(\tau)e^{24\int^\tau f(z)g(z)dz}d\tau$, k_1 is a arbitrary constant and $f(t),\ g(t),\ h(t)$ satisfy the constrain condition

$$f(t)g(t)h'(t)+12f^2(t)g^2(t)h(t)-[f(t)g(t)]'h(t)=0. \tag{8}$$

Using (7), the expression (4) becomes

$$u=w(\xi,\ t),\ \xi=x-\frac{3f(t)g(t)}{h(t)}y^2+c_1(t)y+c_2(t). \tag{9}$$

By using the transformation of variables (9), Equation (5) becomes

$$\frac{\partial}{\partial\xi}[w_t+g(t)(6ww_\xi+w_{\xi\xi\xi})]=f'(t)+12g(t)f^2(t). \tag{10}$$

If

$$f'(t) + 12g(t)f^2(t) = 0, \tag{11}$$

and after integrating Equation (10) with respect to ξ once and taking the constant of integration to zero, Equation (10) becomes the variable coefficient KdV equation (vcKdV) for $w = w(\xi, t)$

$$w_t + g(t)(6ww_\xi + w_{\xi\xi\xi}) = 0. \tag{12}$$

From the discussion above, we come to the conclusion that the $(2+1)$-dimensional gvcKP (1) for $u = u(x, y, t)$ is reduced to the $(1+1)$-dimensional variable coefficient KdV (12) for $w = w(\xi, t)$ by using the transformation of variables (9) under the constrain conditions (8) and (11), if $w(\xi, t)$ is a solution of the variable coefficient KdV (12), substituting it into Equation (9), then we have the exact solution of the gvcKP (1).

3. Solitary Wave Solutions of gvcKP, CKP and vcKP

In previous section, the gvcKP (1) has been reduced into the vcKdV (12) by the transformation (9) under the constrain conditions (8) and (11). The vcKdV (12) is of physically importance and its solutions have been known for many researchers, for instance, according to [15], by means of homogeneous balance priniple the vcKdV (12) has single solitary wave

$$w(\xi, t) = 2s^2 \frac{e^\eta}{(1+e^\eta)^2} + v_0, \ \eta = s\left[\xi - (s^2 + 6v_0)\int^t g(\tau)d\tau + a_0\right], \tag{13}$$

where $s, v_0,$ and a_0 are arbitrary constants. And vcKdV (12) also has 2-soliton solution

$$w(\xi, t) = 2\frac{s_1^2 e^{\eta_1} + s_2^2 e^{\eta_2} + 2(s_1 - s_2)^2 e^{\eta_1+\eta_2} + a_{12}(s_2^2 e^{2\eta_1+\eta_2} + s_1^2 e^{\eta_1+2\eta_2})}{(1 + e^{\eta_1} + e^{\eta_2} + a_{12}e^{\eta_1+\eta_2})^2} + v_0, \tag{14}$$

where s_i and a_i are arbitrary constants, $\eta_i = s_i\left[\xi - (s_i^2 + 6v_0)\int^t g(\tau)d\tau + a_i\right]$, $(i = 1, 2)$, $a_{12} = \dfrac{(s_1 - s_2)^2}{(s_1 + s_2)^2}$.

3.1. Solitary wave solutions of gvcKP

Substituting Equation (13) into transformation (9) we have the single solitary wave solution for gvcKP (1), which is expressed by

$$u(x, y, t) = 2s^2 \frac{e^{\eta}}{(1 + e^{\eta})^2} + v_0, \; \eta = s\left[\xi - (s^2 + 6v_0)\int^t g(\tau)d\tau + a_0\right], \tag{15}$$

where $\xi = x - \dfrac{3f(t)g(t)}{h(t)}y^2 + c_1(t)y + c_2(t)$ in which $c_1(t) = k_1 e^{12\int^t f(\tau)g(\tau)d\tau}$, $c_2(t) = -k_1^2\int^t h(\tau)e^{24\int^\tau f(z)g(z)dz}d\tau$ and $f(t)$, $g(t)$, $h(t)$ satisfy the constrain conditions (8) and (11).

Substituting Equation (14) into transformation (9) we have the 2-solitary solution for gvcKP (1), which is expressed by

$$u(x, y, t) = 2\frac{s_1^2 e^{\eta_1} + s_2^2 e^{\eta_2} + 2(s_1 - s_2)^2 e^{\eta_1+\eta_2} + a_{12}(s_2^2 e^{2\eta_1+\eta_2} + s_1^2 e^{\eta_1+2\eta_2})}{(1 + e^{\eta_1} + e^{\eta_2} + a_{12}e^{\eta_1+\eta_2})^2} + v_0, \tag{16}$$

where s_i and a_i are arbitrary constants, $a_{12} = \dfrac{(s_1 - s_2)^2}{(s_1 + s_2)^2}$, $\eta_i = s_i\left[\xi - (s_i^2 + 6v_0)\int^t g(\tau)d\tau + a_i\right]$, $(i = 1, 2)$, $\xi = x - \dfrac{3f(t)g(t)}{h(t)}y^2 + c_1(t)y$ $+c_2(t)$ in which $c_1(t) = k_1 e^{12\int^t f(\tau)g(\tau)d\tau}$, $c_2(t) = -k_1^2\int^t h(\tau)e^{24\int^\tau f(z)g(z)dz}d\tau$ and $f(t)$, $g(t)$, $h(t)$ satisfy the constrain conditions (8) and (11).

Solitary waves and solitons represent one of the interesting and famous aspects of nonlinear phenomena in spatially extended systems [16].

3.2. Solitary wave solutions of CKP

If $g(t) = 1$, $f(t) = \dfrac{1}{12t}$ and $h(t) = \dfrac{3\alpha^2}{t^2}$, constrain conditions (8) and (11) are satisfied, and solution (15) becomes the single solitary wave solution for CKP (2), which is expressed by

$$u(x, y, t) = 2s^2 \frac{e^{\eta}}{(1+e^{\eta})^2} + v_0, \; \eta = s\left[\xi - (s^2 + 6v_0)t + a_0\right], \tag{17}$$

where $\xi = x - \dfrac{t}{8\alpha^2} y^2 + k_1 ty + 2k_1^2\alpha^2 t$; solution (16) becomes the 2-solitary wave solution for CKP (2), which is expressed by

$$u(x, y, t) = 2\frac{s_1^2 e^{\eta_1} + s_2^2 e^{\eta_2} + 2(s_1 - s_2)^2 e^{\eta_1+\eta_2} + a_{12}(s_2^2 e^{2\eta_1+\eta_2} + s_1^2 e^{\eta_1+2\eta_2})}{(1 + e^{\eta_1} + e^{\eta_2} + a_{12}e^{\eta_1+\eta_2})^2} + v_0, \tag{18}$$

where s_i and a_i are arbitrary constants, $a_{12} = \dfrac{(s_1 - s_2)^2}{(s_1 + s_2)^2}$,

$\eta_i = s_i[\xi - (s_i^2 + 6v_0)t + a_i]$, $(i = 1, 2)$, $\xi = x - \dfrac{t}{8\alpha^2} y^2 + k_1 ty + 2k_1^2\alpha^2 t$.

3.3. Solitary wave solutions of vcKP

If $f(t) = 0$, constrain conditions (8) and (11) are satisfied, and solution (15) becomes the single solitary wave solution for vcKP (3), which is expressed by

$$u(x, y, t) = 2s^2 \frac{e^{\eta}}{(1+e^{\eta})^2} + v_0, \; \eta = s\left[\xi - (s^2 + 6v_0)\int^t g(\tau)d\tau + a_0\right], \tag{19}$$

where $\xi = x + k_1 y - k_1^2 \int^t h(\tau) d\tau$; solution (16) becomes the 2-solitary wave solution for vcKP (3), which is expressed by

$$u(x, y, t) = 2 \frac{s_1^2 e^{\eta_1} + s_2^2 e^{\eta_2} + 2(s_1 - s_2)^2 e^{\eta_1 + \eta_2} + a_{12}(s_2^2 e^{2\eta_1 + \eta_2} + s_1^2 e^{\eta_1 + 2\eta_2})}{(1 + e^{\eta_1} + e^{\eta_2} + a_{12} e^{\eta_1 + \eta_2})^2} + v_0, \tag{20}$$

where s_i and a_i are arbitrary constants, $a_{12} = \frac{(s_1 - s_2)^2}{(s_1 + s_2)^2}$,

$\eta_i = s_i \left[\xi - (s_i^2 + 6v_0) \int^t g(\tau) d\tau + a_i \right]$, $(i = 1, 2)$, $\xi = x + k_1 y - k_1^2 \int^t h(\tau) d\tau$.

4. Conclusion

In this paper, by making corresponding transformation of variables, the $(2 + 1)$-dimensional gvcKP equation is reduced to the $(1 + 1)$-dimensional vcKdV equation, which can be solved by using homogeneous balance method [4, 10, 15] to obtain single solitary wave solution and 2-soliton solution. Substitutiong the solitary solutions of the vcKdV equation into the corresponding transformation of variables, we have the solitary wave solutions of the gvcKP equation. It is interesting to research CKP equation but avoid the singularity point analysis when $t = 0$. The idea of reduction of dimensionality in the present paper may be extended to other works to make further progress.

Acknowledgements

The authors are very grateful to the referees for their invaluable comments.

References

[1] J. K. Xue, Cylindrical and spherical ion-acoustic solitary waves with dissipative effect, Phys. Lett. A 322 (2004), 225-230.

[2] R. S. Johnson, Water waves and Korteweg-de Vries equations, J. Fluid Mech. 97 (1980), 701-709.

[3] R. S. Johnson, A Modern Introduction to the Mathematical Theory of Water Waves, Cambridge University Press, Cambridge, 1997.

[4] Z. Y. Yan, Backlund transformation, non-local symmetry and exact solution for $(2+1)$-dimensional variable coefficient generalized KP equations, Commun. Nonlinear Sci. Numer. Simul. 5 (2000), 31-35.

[5] H. C. Ma, Z. Y. Qin and A. P. Deng, Lie symmetry and exact solution of $(2+1)$-dimensional generalized Kadomtsev-Petviashvili equation with variable coefficients, Therm. Sci. 17(5) (2013), 1490-1493.

[6] A. Pankov and K. Pflüger, On ground-traveling waves for the generalized Kadomtsev-Petviashvili equations, Math. Phys. Anal. and Geom. 3 (2000), 33-47.

[7] M. L. Wang, X. L. Li and J. L. Zhang, Two-soliton solution to a generalized KP equation with general variable coefficients, Appl. Math. Lett. 76 (2018), 21-27. http://dx.doi.org/10.1016/j.aml.2017.07.011.

[8] A. R. Seadawy, Travelling-wave solutions of a weakly nonlinear two-dimensional higher-order Kadomtsev-Petviashvili dynamical equation for dispersive shallow-water waves, Eur. Phys. J. Plus 132 (2017), 29.

[9] S. F. Deng, The decay mode solutions for cylindrical KP equation, Appl. Math. Comput. 218 (2012), 5974-5981.

[10] M. L. Wang, J. L. Zhang and X. Z. Li, Decay model solutions to cylindrical KP equation, Appl. Math. Lett. 62 (2016), 29-34.

[11] X. Z. Li, J. L. Zhang and M. L. Wang, Solitary wave solutions of KP equation, cylindrical KP equation and spherical KP equation, Commun. Theor. Phys. 67 (2017), 207-211.

[12] L. T. Gai, S. Bilige and Y. Jie, The exact solutions and approximate analytic solutions of the $(2+1)$-dimensional KP equation based on symmetry method, Springer Plus 5 (2016), 1267.

[13] N. S. Saini, N. Kaur and T. S. Gill, Dust acoustic solitary waves of Kadomstev-Petviashvili (KP) equation in superthermal dusty plasma, Advan. Space Res. 55 (2015), 2873-2882.

[14] X. Z. Li, J. L. Zhang and M. L. Wang, Decay mode solutions to $(2+1)$-dimensional Burgers equation, cylindrical Burgers equation and spherical Burgers equation, J. Appl. Math. Phys. 5 (2017), 1009-1015.

[15] M. L. Wang and Y. M. Wang, A new Bäcklund transformation and multi-soliton solutions to the KdV equation with general variable coefficients, Phys. Lett. A 287 (2001), 211-216.

[16] A. R. Seadawy, Fractional solitary wave solutions of the nonlinear higher-order extended KdV equation in a stratified shear flow: Part I, Comput. Math. Appl. 70 (2015), 345-352.

■