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OSCILLATORY PROPERTIES OF HIGHER-ORDER DIFFERENTIAL EQUATIONS WITH DISTRIBUTED DELAY

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Abstract

This paper deals with the oscillation properties of higher-order nonlinear differential equations with distributed delay

$$\left[b(t)\left(x^{(n-1)}(t)\right)^{\gamma}\right]' + \int_c^d q(t, \xi)x^{\alpha}(g(t, \xi))d(\xi) = 0, \quad t \geq t_0,$$

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under a condition

$$\int_{t_o}^{\infty} \frac{1}{b^{\frac{1}{\gamma}}(t)} dt < \infty.$$

New oscillation criteria are obtained by employing a refinement of the generalized Riccati transformations and new comparison principles. An example is provided to illustrate the main results.

1. Introduction

In this work, we investigate the oscillation and asymptotic behaviour of solutions to the higher-order nonlinear differential equation with distributed delay of the form

$$\left[b(t)\left(y^{(n-1)}(t)\right)^{\gamma}\right]' + \int_c^d q(t, \xi) y^{\alpha}(g(t, \xi)) d(\xi) = 0, \quad t \geq t_0. \tag{1.1}$$

We assume that the following assumptions hold:

(A_1) $b \in C^1[t_0, \infty)$, $b'(t) \geq 0$, $b(t) > 0$, γ is a quotient of odd positive integers.

(A_2) $q(t, \xi)$, $q(t, \xi) \in C([t_0, \infty) \times [c, d], \mathbb{R})$, $q(t, \xi)$ is positive; $g(t, \xi)$ is nondecreasing function in ξ, $g(t, \xi) \leq t \lim_{t \to \infty} g(t, \xi) = \infty$.

By a solution of Equation (1.1) we mean a function $y \in C^{n-1}[T_y, \infty)$, $T_y \geq t_0$, which has the property $b(t)\left(y^{n-1}(t)\right)^{\gamma} \in C^1[T_y, \infty)$ and satisfies Equation (1.1) on $[T_y, \infty)$. We consider only those solutions y of Equation (1.1) which satisfy $\sup\{|y(t)| : t \geq T\} > 0$, for all $T > T_y$. We assume that (1.1) possesses such a solution. A solution of (1.1) is called oscillatory if it has arbitrarily large zeros on $[T_y, \infty)$ and otherwise it is called to be nonoscillatory. The Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

The problem of the oscillation of higher and fourth order differential equations have been widely studied by many authors, who have provided many techniques for obtaining oscillatory criteria for higher and fourth order differential equations. We refer the reader to the related books (see [1, 6, 15], [11], [13]) and to the papers (see [2], [3]-[10], [14]-[21]). In the following, we present some related results that served as a motivation for the contents of this paper.

Bazighifan [6] consider the oscillatory properties of the higher-order differential equation

$$\left[b(t)\left(y^{(n-1)}(t)\right)^{\gamma}\right]' + q(t)y^{\beta}(\tau(t)) = 0, \quad t \geq t_0,$$

under the conditions

$$\int_{t_0}^{\infty} \frac{1}{b^{\frac{1}{\gamma}}(t)} dt = \infty,$$

and

$$\int_{t_0}^{\infty} \frac{1}{b^{\frac{1}{\gamma}}(t)} d\dot{t} < \infty. \tag{1.2}$$

Elabbasy et al. [7] studied the oscillation behaviour of the higher-order nonlinear differential equation

$$\left[r(t)\left(y^{(n-1)}(t)\right)^{\alpha}\right]' + \sum_{i=1}^{n} q_i(t) f(y(\tau_i(t))) = 0, \quad t \geq t_0.$$

Moaaz et al. [15], Elabbasy et al. [8, 9], and Zhang et al. [21] examined the oscillation of the fourth-order nonlinear delay differential equation

$$\left[r(t)\,(y'''(t))^{\alpha}\right]' + q(t)y^{\alpha}(t) = 0, \quad t \geq t_0.$$

Our aim in the present paper is to employ the Riccatti technique to establish some new conditions for the oscillation of all solutions of Equation (1.1) under the condition (1.2). Some examples are presented to illustrate our main results.

The proof of our main results are essentially based on the following lemmas.

Lemma 1.1 (Baculikova et al. [4]). *If the function z satisfies* $z^{(i)} > 0$, $i = 0, 1, \dots, n,$ *and* $z^{(n+1)} < 0,$ *then*

$$\frac{z(t)}{t^n / n!} \geq \frac{z'(t)}{t^{n-1} / (n-1)!}.$$

Lemma 1.2. (Agarwal et al. [1]). *Let* $z \in (C^n[t_0, \infty], \mathbb{R}^+)$ *and assume that* $z^{(n)}$ *is of fixed sign and not identically zero on a subray of* $[t_0, \infty]$. *If moreover,* $z(t) > 0,\ z^{(n-1)}(t) z^{(n)}(t) \leq 0$ *and* $\lim_{t\to\infty} z(t) \neq 0,$ *then for every* $\lambda \in (0, 1),$ *there exists* $t_\lambda \geq t_0$ *such that*

$$z(t) \geq \frac{\lambda}{(n-1)} t^{n-1} \left| z^{(n-1)}(t) \right|, \text{ for } t \in [t_\lambda, \infty).$$

Lemma 1.3 (Zhang et al. [19]). *Let* $\beta \geq 1$ *be a ratio of two numbers; where U and V are constants. Then*

$$Uy - Vy^{\frac{\beta+1}{\beta}} \leq \frac{\beta^\beta}{(\beta+1)^{\beta+1}} \frac{U^{\beta+1}}{V^\beta}, \ V > 0.$$

2. Main Results

In this section, we shall establish some oscillation criteria for Equation (1.1). We are now ready to state and prove the main results. For convenience, we denote

$$R(s) := \int_{t_0}^{\infty} \frac{1}{b(s)} ds,\ \delta'_+(t) := \max\{0, \delta'(t)\},$$

$$Q(t) = \int_c^d q(t, \xi) d(\xi) \text{ and } \sigma(v) = \int_v^{\infty} Q(s)(g(s, c) \setminus s)^{3\gamma} dv.$$

Theorem 2.1. *Let* $(A_1), (A_2)$ *and* (1.2) *hold. Assume that there exists a positive function* $\delta \in C^1[t_0, \infty)$ *such that*

$$\int_{t_0}^{\infty}\left[\delta(s)\frac{1}{(n-4)!}\int_t^{\infty}(v-s)^{(n-4)}\sigma^{\frac{1}{\gamma}}(v)b(v)^{-1\setminus\gamma}dv + \frac{((\delta'(s))_+)^2}{4\delta(s)}\right]ds = \infty. \quad (2.1)$$

If

$$\int_{t_0}^{\infty}\left[Q(s)\left(\frac{\lambda_2}{(n-2)!}g^{n-2}(s, c)\right)^{\gamma}R^{\gamma}(s) - \left(\frac{\gamma}{\gamma+1}\right)^{\gamma+1}\frac{b^{-1/\gamma}(s)}{R(s)}\right]ds = \infty, \quad (2.2)$$

for some constant $\lambda_2 \in (0, 1)$, *then every solution of* (1.1) *is oscillatory*.

Proof. Assume that (1.1) has a nonoscillatory solution y. Without loss of generality, we can assume that $y(t) > 0$. It follows from (1.1) that there exist two possible cases for $t \geq t_1$, where $t_1 \geq t_0$ is sufficiently large:

Case 1. $y(t) > 0,\ y'(t) > 0,\ y^{(n-1)}(t) > 0,\ y^{(n)}(t) < 0,\ \left(b\left(y^{(n-1)}\right)^{\gamma}\right)'(t) \leq 0.$

Case 2. $y(t) > 0,\ y'(t) > 0,\ y^{(n-2)}(t) > 0,\ y^{(n-1)}(t) < 0,\ \left(b\left(y^{(n-1)}\right)^{\gamma}\right)'(t) \leq 0,$

for $t > t_1$, t_1 is large enough.

Assume that Case 1 holds. From Lemma 1.2, we find $y(t) \geq (t/3)y'(t)$ and, hence

$$\frac{y(g(t, c))}{y(t)} \geq \frac{g^3(t, c)}{t^3}. \quad (2.3)$$

Integrating (1.1) from t to ∞, we obtain

$$-b(t)\left(y^{(n-1)}(t)\right)^{\gamma} \leq -\int_t^{\infty}Q(s)y^{\alpha}(g(s, \xi))d(\xi). \quad (2.4)$$

By virtue of $y'(t) > 0$, $g(t, \xi) \leq t$ and (2.3), we obtain

$$-(y'''(t)) + \frac{y(t)}{b(t)^{1/\gamma}}\left[\int_t^{\infty} Q(s)(g(s, c) \setminus s)^{3\gamma} ds\right]^{1\setminus\alpha} \leq 0. \tag{2.5}$$

Integrating (2.4) from t to ∞ for a total of $(n-3)$-times, we find

$$y''(t) + \frac{y(t)}{(n-4)!}\int_t^{\infty}(v-t)^{(n-4)}\sigma^{\frac{1}{\gamma}}(v)b(v)^{-1\setminus\gamma} dv \leq 0. \tag{2.6}$$

Define the function $\omega(t)$ by

$$\omega(t) := \delta(t)\frac{y'(t)}{y(t)}. \tag{2.7}$$

Then $\omega(t) > 0$ for $t \geq t_1$ and

$$\omega'(t) := \delta'(t)\frac{y'(t)}{y(t)} + \delta(t)\frac{y''(t)y(t) - (y'(t))^2}{y^2(t)}. \tag{2.8}$$

From (2.6) and (2.7), it follows that

$$\omega'(t) \leq -\delta(t)\frac{1}{(n-4)!}\int_t^{\infty}(v-t)^{(n-4)}\sigma^{\frac{1}{\gamma}}(v)b(v)^{-1\setminus\gamma} dv \tag{2.9}$$

$$+\frac{(\delta'(t))_+}{\delta(t)}\omega(t) - \frac{1}{\delta(t)}\omega^2(t).$$

Hence, we have

$$\omega'(t) \leq -\delta(t)\frac{1}{(n-4)!}\int_t^{\infty}(v-t)^{(n-4)}\sigma^{\frac{1}{\gamma}}(v)b(v)^{-1\setminus\gamma} dv + \frac{((\delta'(t))_+)^2}{4\delta(t)}. \tag{4.10}$$

Integrating (2.10) from t_1 to t, we get

$$\int_{t_1}^{t}\left(\delta(s)\frac{1}{(n-4)!}\int_t^{\infty}(v-s)^{(n-4)}\sigma^{\frac{1}{\gamma}}(v)b(v)^{-1\setminus\gamma} dv + \frac{((\delta'(s))_+)^2}{4\delta(s)}\right)ds \leq \omega(t_1),$$

for all large t, which contradicts (2.1).

Assume that Case 2 holds. Noting that $b(t)\left(y^{(n-1)}(t)\right)^{\gamma}$ is nonincreasing, we have that $b(s)\left(y^{(n-1)}(s)\right)^{\gamma} \leq b(t)\left(y^{(n-1)}(t)\right)^{\gamma}$ for all $s \geq t \geq t_1$. This yields

$$y^{(n-1)}(s) \leq \left[b(t)\left(y^{(n-1)}(t)\right)^{\gamma}\right]^{1/\gamma} \frac{1}{b^{1/\gamma}(s)}.$$

Integrating this inequality from t to u, we get

$$y^{(n-2)}(u) - y^{(n-2)}(t) \leq \left[b(t)\left(y^{(n-1)}(t)\right)^{\gamma}\right]^{1/\gamma} \int_t^u \frac{1}{b^{1/\gamma}(s)} ds.$$

Letting $u \to \infty$, we see that

$$-y^{(n-2)}(t) \leq \left[b(t)\left(y^{(n-1)}(t)\right)^{\gamma}\right]^{1/\gamma} R(t). \tag{2.11}$$

From Lemma 1.1, we get

$$y(t) \geq \frac{\lambda}{(n-2)!} t^{n-2} y^{(n-2)}(t), \tag{2.12}$$

for all $\lambda \in (0, 1)$ and every sufficiently large t. Next, we define

$$\varphi(t) = \frac{b(t)\left(y^{(n-1)}(t)\right)^{\gamma}}{\left(y^{(n-2)}(t)\right)^{\gamma}}. \tag{2.13}$$

We note that $\varphi(t) < 0$ for $t \geq t_1$ and

$$\varphi'(t) = \frac{\left(b(t)\left(y^{(n-1)}(t)\right)^{\gamma}\right)'}{\left(y^{(n-2)}(t)\right)^{\gamma}} - \gamma \frac{b(t)\left(y^{(n-1)}(t)\right)^{\gamma+1}}{\left(y^{(n-2)}(t)\right)^{\gamma+1}}.$$

From (1.1) and (2.13), we obtain

$$\varphi'(t) = -Q(t) \frac{y^{\gamma}(g(t, c))}{\left(y^{(n-2)}g(t, c)\right)^{\gamma}} \frac{\left(y^{(n-2)}g(t, c)\right)^{\gamma}}{\left(y^{(n-2)}(t)\right)^{\gamma}} - \gamma \frac{1}{b^{1/\gamma}(t)} \varphi^{\frac{\gamma+1}{\gamma}}(t). \tag{2.14}$$

Hence, (2.14) yields

$$\varphi'(t) \leq -Q(s)\left(\frac{\lambda_2}{(n-2)!}g^{n-2}(s, c)\right)^{\gamma} - \gamma\frac{1}{b^{1/\gamma}(t)}\varphi^{\frac{\gamma+1}{\gamma}}(t). \tag{2.15}$$

Multiplying (2.15) by $R^{\gamma}(t)$ and integrating from t_2 to t, we obtain

$$R^{\gamma}(t)\varphi(t) - R^{\gamma}(t_2)\varphi(t_2) + \gamma\int_{t_2}^{t}\frac{R^{\gamma-1}(s)}{b^{1/\gamma}(s)}\varphi(s)ds$$

$$\leq -\int_{t_2}^{t}Q(s)\left(\frac{\lambda_2}{(n-2)!}g^{n-2}(s, c)\right)^{\gamma}R^{\gamma}(s)ds - \gamma\int_{t_2}^{t}\frac{\varphi^{\frac{1+\alpha}{\alpha}}(s)}{b^{1/\gamma}(s)}R^{\gamma}(s)ds,$$

we set

$$U := \frac{R^{\gamma-1}(s)}{b^{1/\gamma}(s)},\ V := \frac{R^{\gamma}(s)}{b^{1/\gamma}(s)},\ y := -\varphi(s).$$

From Lemma 1.3, we find

$$\int_{t_2}^{t}\left[Q(s)\left(\frac{\lambda_2}{(n-2)!}g^{n-2}(s, c)\right)^{\gamma}R^{\gamma}(s) - \left(\frac{\gamma}{\gamma+1}\right)^{\gamma+1}\frac{b^{-1/\gamma}}{R(s)}\right]ds$$

$$\leq 1 + R^{\gamma}(t_2)\varphi(t_2),$$

for some constant $\lambda_2 \in (0, 1)$, which contradicts (2.2).

Theorem 2.1 is proved. □

It is well known (see [3]) that the differential equation

$$\left[a(t)(y'(t))^{\alpha}\right]' + q(t)y^{\alpha}(\tau(t)) = 0, \quad t \geq t_0, \tag{2.16}$$

where $\alpha > 0$ is the ratio of odd positive integers, $a, q \in C[t_0, \infty)$, is nonoscillatory if and only if there exist a number $T \geq t_0$ and a function $v \in C^1[T, \infty)$, satisfying the inequality

$$v'(t) + \alpha a^{\frac{-1}{\alpha}}(t)(v(t))^{\frac{(1+\alpha)}{\alpha}} + q(t) \leq 0, \quad \text{on } [T, \infty).$$

In what follows, we compare the oscillatory behaviour of (1.1) with the second-order half-linear equations of type (2.16).

Theorem 2.2. *Let* $(A_1), (A_2)$ *and* (1.2) *hold. Assume that the differential equation*

$$\left[\frac{b(t)}{t^{2\gamma}}(y'(t))^{\gamma}\right]' + Q(t)\left(\frac{\lambda_1 g^3(t, c)}{2t^3}\right)^{\gamma} y^{\gamma}(t) = 0, \tag{2.17}$$

is oscillatory for some constant $\lambda_1 \in (0, 1)$, *the equation*

$$\left[b(t)(y'(t))^{\gamma}\right]' + Q(t)\left(\frac{\lambda}{(n-2)!} g^{n-2}(t, c)\right)^{\gamma} y^{\gamma}(t) = 0, \tag{2.18}$$

is oscillatory for some constant $\lambda \in (0, 1)$. *Then every solution of* (1.1) *is oscillatory*.

Proof. Proceeding as in the proof of Theorem 2.1. If we set $\delta(t) = 1$ in (2.10), then we get

$$\omega'(t) + \frac{1}{(n-4)!}\int_t^{\infty}(v-t)^{n-4}\sigma^{\frac{1}{\gamma}}(v)b(v)^{-1\setminus\gamma}dv \leq 0.$$

Thus, we can see that Equation (2.17) is nonoscillatory which is a contradiction. From (2.15), we get

$$\varphi'(t) + Q(s)\left(\frac{\lambda_2}{(n-2)!} g^{n-2}g(s, c)\right)^{\gamma} + \gamma\frac{1}{b^{1/\gamma}(t)}\varphi^{\frac{\gamma+1}{\gamma}}(t) \leq 0,$$

for every constant $\lambda_2 \in (0, 1)$. Thus, we can see that Equation (2.18) is nonoscillatory for every constant $\lambda \in (0, 1)$ which is a contradiction.

Theorem 2.2 is proved. □

3. Example

In this section, we give the following example to illustrate our main results.

Example 3.1. Consider a differential equation

$$\left(t^3(y'''(t))\right)' + \int_0^1 (\nu \setminus t)\xi y\left(\frac{t-\xi}{2}\right)d\xi = 0, \quad t \geq 1, \tag{3.1}$$

where $\nu > 0$ is a constant. Let

$$\gamma = 1,\ n = 4,\ b(t) = t^3 > 0,\ b'(t) = 9t^8 \geq 0,\ b \in C^1[t_0, \infty),$$

$$q(t, \xi) = (\nu \setminus t)\xi > 0,\ q \in C[t_0, \infty),\ c = 0,\ d = 1,$$

$$g(t, c) = \frac{t}{2} \leq t,\ \lim \frac{t}{2} = \infty,\ g(t, c) \in C[t_0, \infty),$$

$$Q(t) = \int_0^1 q(t, \xi)d\xi = \frac{\nu}{t}\int_0^1 \xi d\xi = \frac{\nu}{2t},$$

and hence

$$R(s) := \int_t^\infty \frac{1}{b^{\frac{1}{\gamma}}(s)}ds = \int_t^\infty \frac{1}{s^3}ds = \frac{1}{2s^2}.$$

If we now set $\delta(t) = 1$, then we conclude that (2.1) and (2.2) are satisfied.

$$\int_{t_0}^\infty \left[Q(s)\left(\frac{\lambda_2}{(n-2)!}g^{n-2}(s, c)\right)^\gamma R^\gamma(s) - \left(\frac{\gamma}{\gamma+1}\right)^{\gamma+1}\frac{b^{-1/\gamma}(s)}{R(s)}\right]ds$$

$$= \left(\frac{\nu\lambda_2}{32} - \frac{1}{2}\right)\int_{t_0}^\infty \frac{1}{t}dt = \infty, \text{ if } \nu > \frac{16}{\lambda_2} \text{ for some constant } \lambda_2 \in (0, 1).$$

Hence, by Theorem 2.1, every solution of Equation (3.1) is oscillatory if $\nu > \frac{16}{\lambda_2}$ for some constant $\lambda_2 \in (0, 1)$.

Remark 3.1. The results of [6] cannot confirm the conclusion in Equation (3.1).

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