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CAN A WORMHOLE EXPLAIN THE MISSING DARK MATTER IN THE BULLET CLUSTER?

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Abstract

It has been shown by Brownstein and Moffet that a significant amount of dark matter is missing in the Bullet Cluster 1E0657-558. While this discrepancy can be resolved via a modified gravitational theory, if the dark-matter hypothesis is to be retained, then the possible existence of a wormhole may be the preferred explanation.

1. Introduction

It has been shown that given the Navarro-Frenk-White (NFW) density profile, wormholes could exist in the outer regions of galactic halos [1]. Using the universal rotation curve (URC) dark-matter model, analogous results were obtained for the central part of the halo [2]. The purpose of this note is to show that similar conclusions can be reached by means of the King β-model. Although highly speculative, these ideas deserve to be considered in view of an important finding by Brownstein

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and Moffet [3]: in the Bullet Cluster 1E0657-558, a significant amount of dark matter is missing. Could wormholes provide an explanation?

2. Wormhole Structure

Wormholes are handles or tunnels in spacetime that are able to link widely separated regions of our Universe or different universes in the multiverse model. Such wormholes can be described by the line element

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

using units in which $c = G = 1$. Here $\Phi = \Phi(r)$ is the *redshift function* and $b = b(r)$ is the *shape function*. For the shape function, the defining properties are $b(r_0) = r_0$, where $r = r_0$ is the *throat* of the wormhole. A critical requirement is the *flare-out condition* [4]: $b'(r_0) < 1$, while $b(r) < r$ near the throat.

Before continuing, let us state the Einstein field equations:

$$\rho(r) = \frac{1}{8\pi}\frac{b'}{r^2}, \tag{2}$$

$$p_r(r) = \frac{1}{8\pi}\left[-\frac{b}{r^3} + 2\left(1 - \frac{b}{r}\right)\frac{\Phi'}{r}\right], \tag{3}$$

and

$$p_t(r) = \frac{1}{8\pi}\left(1 - \frac{b}{r}\right)\left[\Phi'' - \frac{b'r - b}{2r(r-b)}\Phi' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r^2(r-b)}\right]. \tag{4}$$

We now recall that the flare-out condition can only be satisfied by violating the null energy condition (NEC), which states that for the stress-energy tensor $T_{\alpha\beta}$,

$$T_{\alpha\beta}k^{\alpha}k^{\beta} \geq 0,$$

for all null vectors k^{α}. To see why, recall that $T_{00} = \rho$ and $T_{11} = p_r$ and consider the outgoing null vector $(1, 1, 0, 0)$. If the NEC is violated, then

$$\rho + p_r < 0.$$

Since $b(r_0) = r_0$ and $b'(r_0) < 1$, it now follows that

$$\rho(r_0) + p_r(r_0) = \frac{1}{8\pi} \frac{b'(r_0) - b(r_0)/r_0}{r_0^2} < 0,$$

as asserted.

3. The King β-Model

When dealing with clusters of galaxies, the total mass is the sum of two parts, the baryonic mass, consisting mainly of intrastellar gas, and the part that is attributable to dark matter or, alternatively, to modified gravity [5]. The baryonic mass has a density given by the King β-model [6]

$$\rho(r) = \rho_0 \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2}, \tag{5}$$

where r_c is the core radius and β and ρ_0 are constants. According to Lobo [5], the total mass is

$$M(r) = \frac{3k_B \beta T}{\mu m_p G} \frac{r^3}{r_c^2 + r^2}, \tag{6}$$

where k_B is Boltzmann's constant, T is the gas temperature, $\mu \approx 0.61$ is the mean atomic weight of the gas particles, and m_p is the proton mass.

At this point we observe that for large r, Equation (6) implies that the total mass inside a sphere of radius r has the linear form

$$M(r) = Cr, \tag{7}$$

that is, the mass increases linearly in the outward radial direction. Since this is exactly what characterizes dark matter, we are justified in assuming that Equation (5) is the approximate density for modelling dark matter. In the other words, purely qualitatively, the dark-matter density has the form

$$\rho(r) = \rho_0\left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2}.$$

Brownstein and Moffet used the value $\beta \approx 0.803$, based on a best-fit determination. Since we are also seeking a qualitative result, we can let $\beta = 2/3$ for computational convenience. Then we get

$$\rho(r) = \rho_0\left(1 + \frac{r^2}{r_c^2}\right)^{-1}. \tag{8}$$

4. A Wormhole in the Bullet Cluster

Using Equation (8), we get from Equation (2)

$$b'(r) = 8\pi\rho_0 \frac{r^2}{1 + (r / r_c)^2}. \tag{9}$$

Since ρ_0 is a small constant, we see at once that the flare-out condition is met, regardless of the location of the throat.

Integrating Equation (9), we obtain

$$b(r) = 8\pi\rho_0 r_c^2\left(r - r_c \tan^{-1}\frac{r}{r_c}\right) + C.$$

To satisfy the condition $b(r_0) = r_0$, we need to evaluate C. The result is

$$b(r) = 8\pi\rho_0 r_c^2\left(r - r_c \tan^{-1}\frac{r}{r_c}\right) + r_0 - 8\pi\rho_0 r_c^2\left(r_0 - r_c \tan^{-1}\frac{r_0}{r_c}\right). \tag{10}$$

So all the conditions for the existence of traversable wormholes have been met. However, to complete the description, some attention should be paid to the redshift function. Here it is sufficient to note that Equation (7) characterizes flat rotation curves due to dark matter [1]. So we can assume that $e^{2\Phi(r)} = Br^{l}$ [1]. Here $l = 2(v^{\phi})^{2}$, where v^{ϕ} is the tangential velocity, and B is an integration constant.

5. Discussion and Summary

As noted in the Introduction, according to Brownstein and Moffet [3], a significant amount of dark matter is missing in the Bullet Cluster 1E0657-558. The main argument in [3] is that this phenomenon can be explained by means of the modified gravity model (MOG) — without the need for dark matter. Unfortunately for this theory, the Bullet Cluster has supposedly shown that dark matter actually exists. So the missing dark matter needs to be explained: an obvious way is to appeal to the possible existence of a wormhole, allowing the dark matter to escape. This may be all the more reasonable because the Bullet Cluster consists of two colliding clusters of galaxies moving at very high velocities, so that the dark matter is literally driven into the wormhole.

In summary, while the proposal in this note is highly speculative, it should not be simply dismissed: if the dark-matter hypothesis is retained and if significant amounts of dark matter are indeed missing in the Bullet Cluster, then the existence of a wormhole can provide a possible explanation.

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