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PARTITIONS OF EVEN NUMBERS AND SOME APPLICATIONS

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Abstract

This article presents the concept of the Partitioned Matrix of an even numbers $w \geq 4$, and a set of formulas for determining the values of the three possible types of partition: odd composite numbers (Cw), prime numbers or Goldbach partitions (Gw) and partitions of mixed numbers, i.e., a prime plus an odd composite in any order (Mw). One of the merits of this study is the set of formulas that completes the fundamental equation of the partitions of an even number $w \geq 4 : Rw = Cw + Gw + Mw$. All the proposed formulas use integral logarithms (Li) and are easily calculable.

1. Introduction

The purpose of this study is improve and apply the concept of the Partitioned Matrix [11].

The approach, if one considers the concepts that it uses, is simple, being nothing more than an algebraic approach that is sufficient to

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demonstrate the proposed formulas and the theorem. It is worth remembering the words of Einstein [3], who claimed that most of the fundamental ideas of science are essentially simple and can, as a rule, be expressed in language that can be understood by everyone.

This study is divided into several parts. Initially some symbols and concepts that are used in the article are addressed. In Section 4, partitions of even numbers $w \geq 4$ are discussed. In Section 5 are presented some applications of the model of partitions here discussed. The work is brought to a close with the conclusions and some recommendations.

2. Symbols and Concepts

Some of the symbols and concepts used in the present study will now be presented.

$\lceil x \rceil$: (i) for any $x \in R$ denotes rounding up.

Cw: (i) for any given even number $w \geq 4$, this is the number of partitions made with odd composite numbers irrespective of order, the sum of which is w; (ii) for example (see Figures 1 and 2), the number $C_{60} = 5$, which means that there are five pairs of odd composite numbers whose sum, irrespective of their order, is $60 : \{(9; 51), (15; 45), (21; 39), (25; 35), (27; 33)\}$; $C_{62} = 1$, $\{(27; 35)\}$; and $C_{64} = 3$, $\{(9; 55), (15; 49), (25; 39)\}$; (iii) the number 1, in that it is not prime, is here considered an odd composite number, on the basis that 1 = 1.1.

ε :(i) is the absolute value of the difference between the observed value and the expected value: is calculated as the difference between the observed value (O) and the expected value (E), i.e., $\varepsilon = |O - E|$; (ii) $|\text{error}|$ is the absolute value of the relative difference between the observed value and the expected value; is calculated as the difference between the observed value (O) and the expected value (E) divided by the expected value (E), i.e., $|\text{error}| = |(O - E) / E|$.

60

	Vector A	Vector B	
1	1	59	m1
2	3	57	m2
3	5	55	m3
4	7	53	g1
5	**9**	**51**	c1
6	11	49	m4
7	13	47	g2
8	**15**	**45**	c2
9	17	43	g3
10	19	41	g4
11	**21**	**39**	c3
12	23	37	g5
13	**25**	**35**	c4
14	**27**	**33**	c5
15	29	31	g6

C60=	5
G60=	6
M60=	4
R60=	15

62

	Vector A	Vector B	
1	1	61	m1
2	3	59	g1
3	5	57	m2
4	7	55	m3
5	9	53	m4
6	11	51	m5
7	13	49	m6
8	15	47	m7
9	17	45	m8
10	19	43	g2
11	21	41	m9
12	23	39	m10
13	25	37	m11
14	**27**	**35**	c1
15	29	33	m12
16	31	31	g3

C62=	1
G62=	3
M62=	12
R62=	16

64

	Vector A	Vector B	
1	**1**	**63**	c1
2	3	61	g1
3	5	59	g2
4	7	57	m1
5	**9**	**55**	c2
6	11	53	g3
7	13	51	m2
8	**15**	**49**	c3
9	17	47	g4
10	19	45	m3
11	21	43	m4
12	23	41	g5
13	**25**	**39**	c4
14	27	37	m5
15	29	35	m6
16	31	33	m7
17	33	31	m8

C64=	4
G64=	5
M64=	8
R64=	17

Figure 1. Partitioned matrices of the numbers 60, 62, and 64 as examples.

Gw: (i) for a given even number $w \geq 4$, this is the number of partitions of prime numbers, irrespective of order, whose sum is w; (ii) is the number of Goldbach partitions; (iii) for example (see Figure 1), the number $G_{60} = 6$, which means that there are 6 pairs of prime numbers, the sum of which, irrespective of their order, is 60 : {(7; 53), (13; 47), (17; 43), (19; 41), (23; 37), (29; 31)}; $G_{62} = 3$, {(3; 59), (19; 43), (31; 31)}; and $G_{64} = 5$, {(3; 61), (5; 59), (11; 53), (17; 47), (23; 41)}.

Rw: (i) for a given even number $w \geq 4$, it is the number of lines (rows) in the partitioned matrix; (ii) the value of Rw is given by

$$Rw = \left\lceil \frac{w}{4} \right\rceil. \tag{1}$$

(iii) For example (see Figure 1), $R_{60} = 15$, $R_{62} = 16$ and $R_{64} = 17$;

Rw$_a$: this is the number of available lines calculated as follows: $Rw_a = Rw - Cw$.

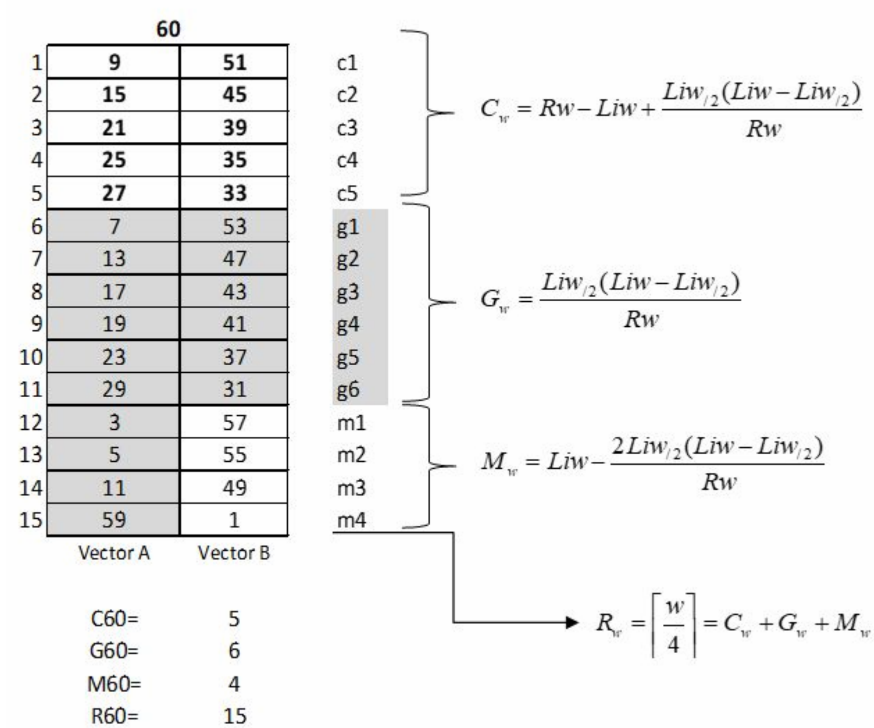

Figure 2. Example of structured partitioned matrix, $w = 60$.

The Partitioned Matrix of the even number $w \geq 4$: (i) the matrix composed of two columns and Rw lines showing the possible partitions of odd composite numbers (c_i), prime numbers (g_i), and mixed numbers (m_i), a prime plus an odd composite or an odd composite plus a prime; (ii) the first column (Vector A) contains the odd numbers arranged in ascending order and the second column (Vector B) has the odd numbers arranged in descending order so that the sum of the partitions in each line is w; (iii) the constant values in the column of Vector A are less than or equal to the constant values in the column of Vector B (see Figures 1 and 3); (iv) the Partitioned Matrix is said to be Structured when the lines are stratified by partitions of odd composite numbers (c_i), prime numbers (g_i), and mixed numbers (m_i) (see example in Figure 2).

Mw: (i) for a given even number $w \geq 4$, this is the number of mixed partitions made with an odd composite number and a prime number, irrespective of order, the sum of which is *w*; (ii) for example (see Figure 1), the number $M_{60} = 4$, which means that there are 4 pairs of a composite number and a prime number whose sum, irrespective of their order, is $60 : \{(1;\ 59),\ (3;\ 57),\ (5;\ 55),\ (11;\ 49)\}$.

p: (i) a natural prime number that has exactly two different divisors: the number one and it self; (ii) the notion of prime number is reserved for natural whole numbers [2], which in the present work will be considered as the set of non-negative whole numbers: $N = N^* \cup \{0\} = \{0, 1, 2, 3, \ldots\}$.

w: (i) an even number of the form $2n$.

Liw: (i) the number of prime numbers that exist up to the number *w* calculated by (3); (ii) in this work which studies partitions of numbers $w \geq 4$, strictly speaking, the prime number 2 should be disregarded in all counts; however, as it is irrelevant to the analysis, the count adopted is not as *Liw*-1; (iii) the value of Liw_g considers the approximation proposed by Gauss [5]:

$$\pi(w) = Liw_g = \int_2^w \frac{dt}{\ln(t)}, \tag{2}$$

Liw_g (2) can be calculated very easily and with very small relative error by the formula *Liw* (3) as shown in Pando [8]. In the calculation of *Liw* by Formula (3), the error tends to be lower for higher values of *w*.

$$Liw = w\left(\frac{0!}{\ln w} + \frac{1!}{\ln^2 w} + \frac{2!}{\ln^3 w} + \frac{3!}{\ln^4 w} + \frac{4!}{\ln^5 w} + \frac{5!}{\ln^6 w}\right) + \varepsilon \in Z_+. \tag{3}$$

$\mathbf{Liw_{/2}}$: (i) the number of prime numbers that exist up to the number $w/2$.

3. Basic Assumption

The concept of the Partitioned Matrix of a number $w \geq 4$ [11] is used, in this study, with two columns and *Rw* lines. These lines are known as (*Gw*) Goldbach partitions (g_i) when the partition is made up of two prime numbers; (*Cw*) composite or odd composite partitions when the partition is made up of two odd composite numbers (c_i); and (*Mw*) mixed partitions (m_i) constituted by a prime number and an odd composite number in any order. Figure 2 provides an example of the Partitioned Matrix for the number 60 in a structured form. In the upper part, we have the partitions of composites $(c_1$ to $c_5)$, followed by the Goldbach partitions $(g_1$ to $g_6)$ and the mixed partitions $(m_1$ to $m_4)$.

4. Partitions of an Even Number ≥ 4

This section is dedicated to proving some lemmas about the characteristics of the partitions of even numbers $w \geq 4$.

Lemma 1. *For every even number* $w \geq 4$, *it is possible to establish the corresponding Partitioned Matrix.*

Proof. For every even number $w \geq 4$, the respective set of partitions can be established, with Vector A of the matrix written with odd number t_1 arranged in ascending order and Vector B of the matrix written with odd numbers t_2 arranged in descending order, so that $w = t_1 + t_2$. See Figure 3, which shows as examples the Partitioned Matrices of the even number from 4 to 14; the Partitioned Matrices of the other numbers w are constructed likewise. It should be remembered that the numbers in the column of Vector A are lower than those in Vector B and can be the same in the last line of the matrix. □

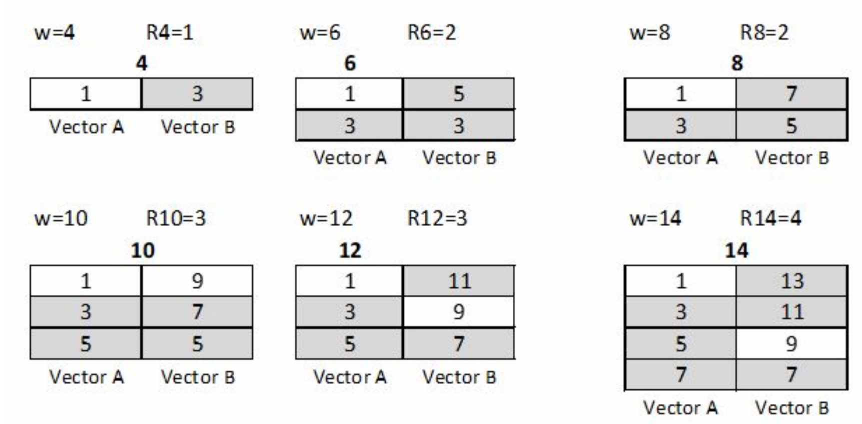

Figure 3. Examples of partitioned matrices for even numbers w from 4 to 14.

Lemma 2. *For any even number* $w \geq 4$, *the Partitioned Matrix has a determined number of lines given by Formula* (1).

Proof. For a given even number $w \geq 4$, the partitions are written as the sum of two odd numbers in such a way that $w = t_1 + t_2$; thus, only half ($w/2$) of the numbers are odd numbers (t_1 and t_2) that are used to establish the partitions of 1 to w. The numbers t_1 and t_2 in turn are distributed in the columns of the matrix (Vectors A and B), so that the number of lines is $(w / 2 / 2 = w / 4)$. As the last line of the Partitioned Matrix of w can be written with the repetition of a number, the number of lines is given by Formula (1) which corresponds to the division of $w / 4$ rounded to the next highest whole when it is not exact. □

Lemma 3. *For any even number* $w \geq 4$ *in the Partitioned Matrix, the number of lines Rw is equal to the sum of the numbers of partitions of odd composite numbers* (*Cw*), *Goldbach partitions* (*Gw*), *and mixed partitions* (*Mw*):

$$Rw = Cw + Gw + Mw. \tag{4}$$

Proof. Each line of a Partitioned Matrix is constituted either by two odd composite numbers (c_w) or by two prime numbers (g_w); if it is neither of these two cases, the line can only be constituted by an odd composite number and a prime number, in any order, with no other possibility available. Thus: (4) $Rw = Cw + Gw + Mw$. See examples in Figure 1. □

For any Partitioned Matrix of an even number $w \geq 4$:

$$Gw = Liw - (Rw - Cw). \tag{5}$$

In any Partitioned Matrix of an even number $w \geq 4$, the number of lines available Rw_a to contain the number of prime numbers *Liw* is given by

$$Rw_a = Rw - Cw. \tag{6}$$

This concept can be viewed with the aid of Figure 2: the available lines Rw_a, in this case, are the sixth to the fifteenth, which represents $Rw_a(60) = 10$. Since $Liw = 16$, there are more prime numbers than lines available to "receive" them, thus $G(60) = 6$.

Lemma 4. *The Partitioned Matrix of an even number* $w \geq 4$ *contains a certain number of partitions Cw, Gw, and Mw derived from Formula* (4).

Proof. It is known that the amount of prime numbers associated with a *w* value are distributed among the partitions *Gw* and *Mw* such that

$$Liw = 2Gw + Mw. \tag{7}$$

From here is derived:

$$Mw = Liw - 2Gw. \tag{8}$$

Replacing:

$$Rw = Cw - Gw + Liw; \tag{9}$$

$$Cw = Rw + Gw - Liw. \tag{10}$$

From Formula (7) can be derived

$$Liw - Liw_{/2} = 2Gw + Mw - Liw_{/2}; \tag{11}$$

$$\frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} = \frac{(2Gw + Mw - Liw_{/2})Liw_{/2}}{Rw}. \tag{12}$$

As illustrated in Figure 4, Formula (12) corresponds to:

$$Gw = Liw_{/2} \frac{(Liw - Liw_{/2})}{Rw}. \tag{13}$$

By substituting (13) in the formulas (8) and (10), we have:

$$Mw = Liw - 2\frac{Liw_{/2}(Liw - Liw_{/2})}{Rw}; \tag{14}$$

$$Cw = Rw - Liw + \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw}. \tag{15}$$

Formula (13) provides more accurate values of Formula (16) that is in accordance with Sylvester [10] with the adjustment proposed by Legendre [6]:

$$Gw_{Syl} = \frac{w}{(\ln w - 1.08366)^2}. \tag{16}$$

Lemma 5. *The relationship* $(Liw - Liw_{/2})/Liw_{/2}$ *tends towards* 1 *when* $w \to \infty$.

Proof. The relationship $(Liw - Liw_{/2})/Liw_{/2}$ tends towards 1 when $w \to \infty$ (see Formula 17).

$$\lim_{w\to\infty} \frac{(Liw - Liw_{/2})}{Liw_{/2}} = \frac{2Liw_{/2} - Liw_{/2}}{Liw_{/2}} = 1. \tag{17}$$

It is known [8] that:

$$\lim_{w\to\infty} Liw \approx \frac{w}{\ln w}. \tag{18}$$

Then:

$$\lim_{w\to\infty}\frac{\left(\frac{w}{\ln w}-\frac{w_{/2}}{\ln w_{/2}}\right)}{\frac{w_{/2}}{\ln w_{/2}}}=\frac{\left(\frac{2w_{/2}}{\ln 2w_{/2}}-\frac{w_{/2}}{\ln w_{/2}}\right)}{\frac{w_{/2}}{\ln w_{/2}}}=\frac{\frac{w_{/2}}{\ln w_{/2}}}{\frac{w_{/2}}{\ln w_{/2}}}=1. \tag{19}$$

□

5. Some Applications of Partitions of Even Numbers

The instruments developed so far, especially the concept of a Partitioned Matrix and Formulas (4), (13), (14), and (15) are adequately accurate and can be applied to some theorems.

Theorem 1. *A demonstration of Goldbach's Conjecture is only necessary for even numbers* $w \geq 98$.

Proof. It is known that from (5) $Gw = Liw - Rw + Cw$. Assuming the partitions of odd composite numbers $Cw = 0$, we have

$$\underset{\to Cw=0}{Gw} = Liw - Rw = \pi(w) - Rw. \tag{20}$$

Thus, the possibility of Goldbach's Conjecture not being achieved occurs from the even number $w = 98$, onwards when the number of $\pi(w)$ is for the first time lower than the number of lines Rw of the Partitioned Matrix. The prime number 2 is not considered, as it is not part of any Partitioned Matrix. In other words, the negative logical conditions that tests whether $\pi(w) \geq Rw$ only occurs when $w = 98$: up to this number, for $w \geq 4$, the number of primes $\pi(w)$ is greater than the number of lines of the Partitioned Matrix. □

Theorem 2. *For* $w \to \infty$, *there are as many prime numbers between* 1 *and w as between w and* 2*w*.

Proof. Above it was seen (Lemma 5) that the relationship $(Liw - Liw_{/2})/Liw_{/2}$ tends towards 1 when $w \to \infty$. This means that for large numbers there tends to be as many primes in Vector A as in Vector B $((Liw - Liw_{/2}) = (Liw_{/2}),$ Figure 4), i.e., in other words, for

$n \to \infty$ there are as many prime numbers between 1 and w as between w and $2w$. This result is in keeping with Loo [7], proving that when n tends towards infinity, the number of primes between $3n$ and $4n$ also tends towards infinity.

□

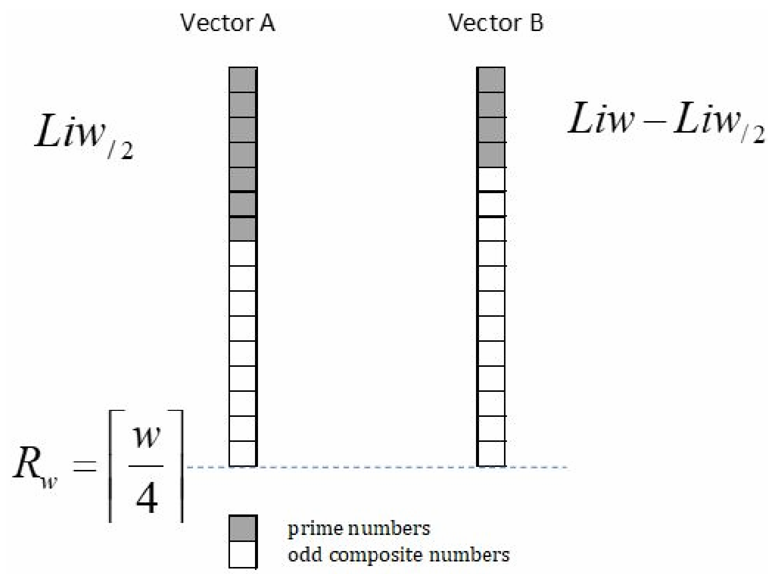

Figure 4. Structured partitioned matrix.

Theorem 3. *For every even number* $w \geq 4$, *there is a certain number of partitions of composite numbers* (Cw), *so that the total number of available lines in the Partitioned Matrix is lower than the number of prime numbers* $\pi(w)$, *which leads to the existence of Goldbach partitions* (Gw), *i.e.*, $(Rw - Cw) < \pi(w)$.

Proof. We have seen that, for a given even number $w \geq 4$, the number of lines in the Partitioned Matrix is

$$Rw = \left\lceil \frac{w}{4} \right\rceil.$$

Formula (15) adequately expresses the number of partitions of odd composite numbers in a Partitioned Matrix of a given even number $w \geq 4$,

$$Cw = Rw - Liw + \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw}.$$

We start from:

$$(Rw - Cw) < \pi(w). \tag{21}$$

Substituting Formulas (4) and (15) in Formula (21), we have:

$$Rw - \left(Rw - Liw + \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw}\right) < Liw; \tag{22}$$

$$Liw - \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} < Liw. \tag{23}$$

For any value w, Formula (23) is true, then Formula (21) is also true: $(Rw - Cw) < \pi(w)$, i.e., the total of the available lines in the Partitioned Matrix is lower than the number of prime numbers $\pi(w)$, which leads to the existence of Goldbach partitions (Gw). This is proof of Goldbach's Conjecture. □

Theorem 4. *It is impossible to observe $Gw = 0$. Therefore, for any even number $w \geq 4$, w is equal to the sum of two prime numbers.*

Proof. The demonstration is performed using an argument of *reductio ad absurdum*, beginning with the hypothesis that there is at least one even number $w \geq 4$ with $Gw = 0$.

In this case, we may suppose that $Gw = 0$:

$$Gw = 0 \to \underset{w \to \infty}{Gw} = Liw_{/2} \frac{Liw - Liw_{/2}}{Rw} = 0. \tag{24}$$

In this case, $Liw_{/2} > 0$ and $Rw > 0$ for any value w (see Figure 4).

$$\underset{w \to \infty}{Gw} = Liw_{/2} \frac{Liw - Liw_{/2}}{Rw} \to Liw_{/2} \frac{0}{Rw} \to (Liw - Liw_{/2}) = 0. \tag{25}$$

As shown in (25), for $Gw = 0$, then $(Liw - Liw_{/2}) = 0$ which implies $Liw = Liw_{/2}$. This condition is shown in Figure 4 where the Vector B must contain zero prime numbers: $(Liw - Liw_{/2}) = 0$.

Thus, for Formula (13) for large numbers tends to be

$$\underset{w \to \to \infty}{Gw} = Liw_{/2} \frac{Liw - Liw_{/2}}{Rw} = Liw_{/2} \frac{2Liw_{/2} - Liw_{/2}}{Rw}$$

$$= \frac{(Liw_{/2})^2}{\frac{w}{4}} = \frac{4(Liw_{/2})^2}{w}. \tag{26}$$

Note that the difference between the values of the two formulas (13) and (26) decreases as w increases tending to zero when $w \to \infty$, which means that the two formulas (13) and (26) are equivalent to large values. With this in mind, one can prove that Formulas (13) and (26) are equivalent: when $w \to \infty$:

$$\underset{w \to \infty}{Gw} = Liw_{/2} \frac{Liw - Liw_{/2}}{Rw} = \frac{4(Liw_{/2})^2}{w}; \tag{27}$$

$$\underset{w \to \infty}{Gw} = \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} = \frac{(Liw_{/2})^2}{\frac{w}{4}}; \tag{28}$$

$$\underset{w \to \infty}{Gw} = \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} = \frac{(Liw_{/2})^2}{Rw}; \tag{29}$$

$$\lim_{w \to \infty} = Liw_{/2}(Liw - Liw_{/2}) = (Liw_{/2})^2. \tag{30}$$

It has been demonstrated by Formulas (17) to (19) that Liw tends to $2Liw_{/2}$ when $w \to \infty$. We can replace Liw by $2Liw_{/2}$ in Formula (30):

$$\lim_{w \to \infty} = Liw_{/2}(2Liw_{/2} - Liw_{/2}) = (Liw_{/2})^2; \tag{31}$$

$$\lim_{w \to \infty} = Liw_{/2}(Liw_{/2}) = (Liw_{/2})^2. \tag{32}$$

Thus, it is impossible that, for any value of w, $Gw = 0$ is observed. Therefore, for any even number $w \geq 4$ is equal to the sum of two prime numbers, observing Formula (4).

Bertrand [2] postulated that for every $n > 1$ there is always at least one prime p such that of $n < p < 2n$. This conjecture has been demonstrated over time and some works on the subject are well known. For example, Ramanujan [9] used properties of the Gamma function to give a simpler proof; Bachraoui [1] proved that there exists a prime between $2n$ and $3n$, Erdös [4] using the Chebyshev function and Loo [7] proved that the number of primes between $3n$ and $4n$ goes to infinity.

This theorem, which shows that for $w \to \infty$ there are as many prime numbers between 1 and w as between w and $2w$ generalizes proof Loo [7]. This is a proof of Goldbach's Conjecture. □

Theorem 5. *Every even number $w \geq 4$ is equal to the sum of two prime numbers.*

Proof. The demonstration is performed using a *reductio ad absurdum* argument, beginning with the hypothesis that there is at least one even number $w \geq 4$ with $Gw = 0$.

In this case, we may suppose that $Gw = 0$, then $Mw = \pi(w) = Liw$. In effect, it can be supposed that there exist **i** cases of partitions of odd composites (Cw) but, as every number cannot have a partition with any other prime, the remaining lines would be constituted by mixed partitions Mw (a prime number and an odd composite in any order).

Therefore, the occurrence of $G_w = 0$ implies that

$$Cw + Mw = Rw \to Gw = 0, \tag{33}$$

where

$$Cw = Rw - Liw + \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw};$$

$$Mw = Liw - \frac{2Liw_{/2}(Liw - Liw_{/2})}{Rw},$$

and in this case:

$$\left(Rw - Liw + \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw}\right) + \left(Liw - \frac{2Liw_{/2}(Liw - Liw_{/2})}{Rw}\right) = Rw; \tag{34}$$

$$Rw - Liw + \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} + Liw - 2\frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} = Rw; \tag{35}$$

$$Rw - \frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} = Rw. \tag{36}$$

The Formula (36) is false, implying:

$$\frac{Liw_{/2}(Liw - Liw_{/2})}{Rw} = 0. \tag{37}$$

This result is very far from zero within some acceptable margin of error, which means there cannot be an even number $w \geq 4$ with $Gw = 0$, i.e., with no partitions of prime numbers. Here, in Formula (37), we apply the same argument used in Formulas (25) to (32). Thus, the initial hypothesis that there is at least one case where $Gw = 0$ is false and, thus it can be concluded that for every $w \geq 4$ exist $Gw > 0$, which proves that Goldbach's Conjecture is true. □

6. Conclusions and Recommendations

The initial concept for this work was the Partitioned Matrix of an even number $w \geq 4$. It was shown that for every even number $w \geq 4$, it is possible to establish a corresponding Partitioned Matrix with a certain number of lines given by Formula (1).

It was proved that, fundamentally, the sum of the partitions is equal to the number of lines in the matrix (4): $Rw = Cw + Gw + Mw.$

It was also shown that for each and every Partitioned Matrix of an even number $w \geq 4$, $Gw = \pi(w) - (Rw - Cw)$, which means that the number of Goldbach partitions or partitions of prime numbers of an even number $w \geq 4$ is given by the number of prime numbers up to w minus the number of available lines (Rw_a), which was calculated as follows: $Rw_a = Rw - Cw.$

The proposed Formulas are (13), (14), and (15).

Formula (13) expresses the number of prime partitions or Goldbach partitions (Gw). Formula (13) is adjusted to Formula (16) by Sylvester and is more accurate.

Formula (14) is innovative and concerned with a new concept, which is that of mixed partitions (Mw), partitions of even numbers $w \geq 4$ constituted by a prime number and an odd composite number in any order.

Formula (15) is also innovative and it refers to partitions of odd composite numbers (Cw), an important concept within Formula (4) $Rw = Cw + Gw + Mw.$ The partitions of odd composite numbers (Cw) are as important as the partitions of prime numbers or Goldbach partitions (Gw). The number of partitions Cw is fundamental for defining the available lines (Rw_a) in a Partitioned Matrix to explain the existence of Gw or Goldbach partitions.

All the proposed formulas Cw, Gw, and Mw make use of easily calculable natural logarithms, using Formula (3) providing values that are very close to the real values.

With the developed instrument, there is a possibility of establishing a proof of Goldbach's Conjecture (every even number $w \geq 4$ is equal to the sum of two prime numbers) and the generalization of proof Loo [7] with reference to Bertrand's Postulate [2].

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