

REAL HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE SATISFYING CERTAIN CONDITIONS

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Abstract

The objective of the present paper is to study real hyper surfaces of a complex projective space with generalized recurrent second fundamental tensor and it is shown that such type real hyper surface exist. Also, we study real hyper surfaces of a complex projective space with generalized recurrent Ricci tensor. It is proved that a real hyper surfaces of complex projective space is generalized Ricci recurrent.

1. Introduction

A Riemannian manifold of constant sectional curvature is called real space form. A complex n -dimensional Kähler manifold of constant holomorphic sectional curvature c is called a complex space form. A

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complete and simply connected complex space form is a complex Euclidean space C^n , if $c = 0$, a complex projective space CP^n , if $c > 0$ or a complex hyperbolic space CH^n , if $c < 0$.

Let CP^n , $n \geq 2$ be an n -dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 4 and let M be a real hyper surface of CP^n . Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the Kähler structure of CP^n . Many differential geometers have been studied real hypersurfaces of a complex projective space such as Bejancu and Deshmukh [1], Cecil and Ryan [2], Cho and Ki [4], Deshmukh [5], Hamada [8, 9, 10], Ikuta [13], Kimura [15, 16, 17, 18], Kimura and Maeda [19], Maeda [21, 22], Maeda [23], Matsuyama [24], [25], [26], Niebergall and Ryan [27], Okumura [28], Perez et al. [30, 31, 32, 33], Takagi [35, 36, 37], Wang [38] and others.

It is well known that there does not exist a real hyper surface M of CP^n satisfying the condition that the second fundamental tensor A of M is parallel. Again in [9], Hamada used the condition that the second fundamental tensor A is recurrent, i.e., there exists an 1-form α such that $\nabla A = A \otimes \alpha$. And Hamada [9] proved that there are no real hyper surfaces of a complex projective space with recurrent second fundamental tensor. In this connection, it is mentioned that Hui and Matsuyama [11] studied real hyper surfaces of a complex projective space with pseudo parallel second fundamental tensor. Also, Hui and Matsuyama [12] studied pseudo Ricci symmetric real hyper surfaces of a complex projective space.

Motivated by the above studies, the present paper deals with the study of real hyper surfaces of a complex projective space with generalized recurrent second fundamental tensor. In this paper, we consider the condition that the second fundamental tensor is generalized recurrent, i.e., there exists two 1-forms α and β such that

$$(\nabla_X A)Y = \alpha(X)AY + \beta(X)Y, \quad (1.1)$$

where ρ and σ are vector fields associated to the 1-forms α and β such that $\alpha(X) = g(X, \rho)$ and $\beta(X) = g(X, \sigma)$. The paper is organized as follows. Section 2 is concerned with some preliminaries. Section 3 is devoted to the study of real hyper surfaces of a complex projective space with generalized recurrent second fundamental tensor. In [9], Hamada proved that there are no real hyper surfaces of a complex projective space with recurrent second fundamental tensor. However, in this paper, we obtain that the associated 1-forms of real hyper surfaces of a complex projective space CP^n with generalized recurrent second fundamental tensor.

A Riemannian space is said to be Ricci symmetric if its Ricci tensor S of type $(0, 2)$ satisfies $\nabla S = 0$, where ∇ denotes the Riemannian connection. During the last five decades, the notion of Ricci symmetry has been weakened by many authors in several ways to a different extent such as Ricci-recurrent space [29], Ricci semisymmetric space [34], pseudo Ricci symmetric space by Deszcz [7], pseudo Ricci symmetric space by Chaki [3]. Generalizing the notion of Ricci-recurrent manifold, De et al. [6] introduced the notion of generalized Ricci-recurrent manifolds. A Riemannian manifold is called a generalized Ricci-recurrent [6] if its Ricci tensor S of type $(0, 2)$ satisfies the condition

$$(\nabla_X S)(Y, Z) = \alpha(X)S(Y, Z) + \beta(X)g(Y, Z), \quad (1.2)$$

where α and β are two non-vanishing 1-forms defined by $\alpha(X) = g(X, \rho_1)$, $\beta(X) = g(X, \rho_2)$.

The relation (1.2) can be written as

$$(\nabla_X Q)(Y) = \alpha(X)QY + \beta(X)Y, \quad (1.3)$$

where Q is the Ricci-operator, i.e., $g(QX, Y) = S(X, Y)$ for all X, Y .

Many differential geometers studied real hyper surfaces of complex projective space satisfying some condition of Ricci tensor. In [14], Ki proved that there are no real hyper surfaces of a complex projective space CP^n with parallel Ricci tensor. Again in [10], Hamada studied real hyper surfaces of a complex projective space with recurrent Ricci tensor and obtained that there are no real hypersurfaces of a complex projective space with recurrent Ricci tensor under the condition that ξ is a principal curvature vector. Also, Loo [20] studied real hyper surfaces in a complex space form with recurrent Ricci tensor.

Motivated by the above studies in the Section 4, we have studied the real hyper surface of a complex projective space with generalized recurrent Ricci tensor, i.e., the Ricci-operator Q satisfies the condition (1.3). Section 4 is devoted to the study of real hyper surfaces of a complex projective space with generalized recurrent Ricci tensor. It is proved that a real hyper surface of a complex projective space can be generalized Ricci recurrent.

2. Preliminaries

Let M be a real hyper surface of CP^n , $n \geq 2$. In a neighbourhood of each point, we take a unit normal vector field N in CP^n . The Riemannian connections $\tilde{\nabla}$ in CP^n and ∇ in M are related by

$$\tilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \quad (2.1)$$

$$\tilde{\nabla}_X N = -AX, \quad (2.2)$$

for arbitrary vector fields X and Y on M , where g is the Riemannian metric of M induced from the Fubini-Study metric G of CP^n and A is the

second fundamental tensor of M in CP^n . Let TM be the tangent bundle of M . An eigenvector X of the second fundamental tensor A is called a principal curvature vector. Also an eigenvalue λ of A is called a principal curvature. It is known that M has an almost contact metric structure induced from the Kähler structure J on CP^n , that is, we define a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η on M by $g(\phi X, Y) = G(JX, Y)$ and $g(\xi, X) = \eta(X) = G(JX, N)$. Then we have

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = g(\xi, \xi) = 1, \quad \phi\xi = 0. \quad (2.3)$$

Also it follows from (2.1) that

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi, \quad (2.4)$$

$$\nabla_X \xi = \phi AX. \quad (2.5)$$

Let \tilde{R} and R be the curvature tensors of CP^n and M , respectively. From the expression of the curvature tensor of CP^n , the curvature tensor, Codazzi equation and the Ricci tensor of type (1,1) are given by

$$\begin{aligned} R(X, Y)Z &= g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y \\ &\quad - 2g(\phi X, Z)\phi Z + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \quad (2.6)$$

$$(\nabla_X A)Y - (\nabla_Y A)X = \eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi, \quad (2.7)$$

$$QX = (2n + 1)X - 3\eta(X)\xi + hAX - A^2 X, \quad (2.8)$$

where $h = \text{trace } A$.

Again we have

$$\begin{aligned} (\nabla_X Q)Y &= -3g(\phi AX, Y)\xi - 3\eta(Y)\phi AX + (Xh)AY \\ &\quad + h(\nabla_X A)Y - A(\nabla_X A)Y - (\nabla_X A)AY. \end{aligned} \quad (2.9)$$

Also we recall the following:

Lemma 2.1 ([23]). *If ξ is a principal curvature vector, then the corresponding principal curvature a is locally constant.*

Lemma 2.2 ([23]). *Assume that ξ is a principal curvature vector and the corresponding principal curvature is a . If $AX = \lambda X$ for $X \perp \xi$, then we have $A\phi X = \bar{\lambda}$, where $\bar{\lambda} = \frac{(a\lambda + 2)}{(2\lambda - a)}$.*

Theorem 2.1 ([2]). *Let M be a connected real hyper surface of CP^n , $n \geq 3$, whose Ricci tensor S satisfies $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$ for some smooth functions a and b on M . Then M is locally congruent to one of the following:*

(i) *a geodesic hyper surface;*

(ii) *a tube of radius r over a totally geodesic CP^k , $1 \leq k \leq n - 2$, where $0 < r < \frac{\pi}{2}$ and $\cot^2 r = \frac{k}{n - k - 1}$;*

(iii) *a tube of radius r over a complex quadric Q_{n-1} , where $0 < r < \frac{\pi}{4}$ and $\cot^2 2r = n - 2$.*

Theorem 2.2 ([35]). *Let M be a homogeneous real hyper surface of CP^n . Then M is a tube of radius r over one of the following Kähler submanifolds:*

(A₁) *hyperplane CP^{n-1} , where $0 < r < \frac{\pi}{2}$;*

(A₂) *totally geodesic CP^k , ($1 \leq k \leq n - 2$), where $0 < r < \frac{\pi}{2}$;*

(B) *complex quadric Q_{n-1} , where $0 < r < \frac{\pi}{4}$;*

(C) *$CP^1 \times CP^{\frac{n-1}{2}}$, where $0 < r < \frac{\pi}{4}$ and $n(\geq 5)$ is odd;*

(D) complex Grassman $cG_{2,5}$, where $0 < r < \frac{\pi}{4}$ and $n = 9$;

(E) Hermitian symmetric space $SO(10)/U(5)$, where $0 < r < \frac{\pi}{4}$ and $n = 15$.

Theorem 2.3 ([16]). *Let M be a real hyper surface of CP^n . Then M has constant principal curvatures and ξ is a principal curvature vector if and only if M is locally congruent to a homogeneous real hyper surface.*

3. Real Hypersurfaces of CP^n with Generalized Recurrent Second Fundamental Tensor

In this section, we study real hyper surfaces of CP^n , $n \geq 2$ with generalized recurrent second fundamental tensor and obtain the following:

Theorem 3.1. *In a real hyper surface of a complex projective space CP^n , $n \geq 2$ with generalized recurrent second fundamental tensor, the associated 1-forms α and β are given in (3.21).*

Proof. We now consider a complex projective space CP^n , $n \geq 2$ with generalized recurrent second fundamental tensor, that is, a complex projective space CP^n whose second fundamental tensor satisfies the Equation (1.1).

Let us take

$$A\xi = a\xi + bU, \quad (3.1)$$

for two functions a and b , where U is the unit tangent vector field orthogonal to ξ on M .

So from (1.1) and (3.1), we have

$$\begin{aligned} g((\nabla_X A)\xi, Y) &= \alpha(X)g(A\xi, Y) + \beta(X)g(\xi, Y) \\ &= [\alpha\alpha(X) + \beta(X)]\eta(Y) + b\alpha(X)g(U, Y), \end{aligned} \quad (3.2)$$

for any tangent vector fields X and Y on M .

Again by virtue of (2.3), we have from (2.7) that

$$(\nabla_X A)\xi = (\nabla_\xi A)X - \phi X. \quad (3.3)$$

Also from (1.1), we get

$$(\nabla_\xi A)X = \alpha(\xi)AX + \beta(\xi)X. \quad (3.4)$$

By virtue of (3.4), it follows from (3.3) that

$$g((\nabla_X A)\xi, Y) = \alpha(\xi)g(AX, Y) + \beta(\xi)g(X, Y) - g(\phi X, Y). \quad (3.5)$$

From (3.2) and (3.5), we have

$$\begin{aligned} \alpha(\xi)g(AX, Y) &= g(\phi X, Y) + [\alpha\alpha(X) + \beta(X)]\eta(Y) \\ &\quad + b\alpha(X)g(U, Y) - \beta(\xi)g(X, Y), \end{aligned} \quad (3.6)$$

for arbitrary vector fields X and Y on M .

Putting $X = \xi$ and $Y = U$ in (3.6), we have

$$\alpha(\xi)g(A\xi, U) = b\alpha(\xi). \quad (3.7)$$

Similarly setting $X = U$ and $Y = \xi$ in (3.6), we get

$$\alpha(\xi)g(AU, \xi) = \alpha\alpha(U) + \beta(U). \quad (3.8)$$

Since the second fundamental tensor is symmetric, we get from (3.7) and (3.8) that

$$\alpha\alpha(U) + \beta(U) = b\alpha(\xi). \quad (3.9)$$

Again putting $X = U$, $Y = \phi U$ in (3.6), we obtain

$$\alpha(\xi)g(AU, \phi U) = g(\phi U, \phi U) = 1. \quad (3.10)$$

Similarly putting $X = \phi U$, $Y = U$ in (3.6) and using (2.3), we have

$$\alpha(\xi)g(A\phi U, U) = b\alpha(\phi U) - 1. \quad (3.11)$$

From (3.10) and (3.11), we have

$$b\alpha(\phi U) = 2. \quad (3.12)$$

Also putting $X = Y = \phi U$ in (3.6), we obtain

$$\alpha(\xi)g(A\phi U, \phi U) = -\beta(\xi), \quad (3.13)$$

which implies that

$$b\alpha(\xi)g(A\phi U, \phi U) = -b\beta(\xi). \quad (3.14)$$

By virtue of (3.9), we have from (3.14) that

$$[\alpha(U) + \beta(U)]g(A\phi U, \phi U) + b\beta(\xi) = 0. \quad (3.15)$$

Now from (1.1), it follows that

$$\begin{aligned} (\nabla_U A)\phi U - (\nabla_{\phi U} A)U &= \alpha(U)A\phi U - \alpha(\phi U)AU \\ &\quad + \beta(U)\phi U - \beta(\phi U)U. \end{aligned} \quad (3.16)$$

Also from Codazzi equation (2.7), we get

$$(\nabla_U A)\phi U - (\nabla_{\phi U} A)U = -2\xi, \quad (3.17)$$

since U is the unit tangent vector field orthogonal to ξ on M . Using (3.17) in (3.16), we get

$$\alpha(U)A\phi U = \alpha(\phi U)AU - \beta(U)\phi U + \beta(\phi U)U - 2\xi,$$

which implies that

$$\alpha(\phi U)g(AU, \phi U) = \alpha(U)g(A\phi U, \phi U) + \beta(U). \quad (3.18)$$

In view of (3.10) and (3.14), (3.18) yields

$$\alpha(\phi U) = \alpha(\xi)\beta(U) - \beta(\xi)\alpha(U) \quad \text{as } \alpha(\xi) \neq 0. \quad (3.19)$$

From (3.12) and (3.19), we get

$$\alpha(\xi)\beta(U) - \beta(\xi)\alpha(U) = \frac{2}{b}. \quad (3.20)$$

From (3.9) and (3.20), we get

$$\alpha(U) = \frac{b^2\{\alpha(\xi)\}^2 - 2}{b[a\alpha(\xi) + \beta(\xi)]} \quad \text{and} \quad \beta(U) = \frac{2a + b^2\alpha(\xi)\beta(\xi)}{b[a\alpha(\xi) + \beta(\xi)]}, \quad (3.21)$$

for any tangent vector field orthogonal to ξ .

4. Real Hypersurfaces of a Complex Projective Space

CP^n with Generalized Recurrent Ricci Tensor

In this section, we have studied real hyper surfaces of a complex projective space CP^n , $n \geq 2$ with generalized recurrent Ricci tensor and prove the following:

Lemma 4.1. *Let M be a connected real hyper surface of a complex projective space CP^n , $n \geq 2$ with generalized recurrent Ricci tensor. If all eigenvalues of the Ricci operator Q are constant, then the Ricci tensor S of M is not parallel.*

Proof. Let λ be an eigenvalue of the Ricci operator corresponding to the unit eigenvector Y . Then we have

$$g((\nabla_X Q)Y, Y) = g(\nabla_X QY, Y) - g(Q(\nabla_X Y), Y) = X\lambda, \quad (4.1)$$

for any X on M .

Again we have from (1.3) that

$$\begin{aligned} g((\nabla_X Q)Y, Y) &= \alpha(X)g(QY, Y) + \beta(X)g(Y, Y) \\ &= \alpha(X)\lambda + \beta(X). \end{aligned} \tag{4.2}$$

Since all the eigenvalues of Q are constant, we get from (4.1) and (4.2) that $\alpha(X)\lambda + \beta(X) = 0$ for all X on M . Consequently, the Ricci tensor S of M is not parallel. From (2.8) and since ξ is principal, the principal curvature vector will also be eigenvectors of S . Thus Ricci tensor of a homogeneous real hyper surface has constant eigenvalues. Again, the hyper surface listed in Theorem 2.2 do not have parallel Ricci tensor. Thus from Lemma 4.1 and Theorem 2.3, we may state the following:

Proposition 4.1. *A homogeneous real hyper surface of CP^n , $n \geq 2$ can be generalized Ricci recurrent.*

So by using Theorem 2.1, we have

Corollary 4.1. *A real hyper surface of CP^n , $n \geq 2$, whose Ricci tensor S satisfies $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$ for some smooth functions a and b on M , can be generalized Ricci recurrent.*

Now we prove the following:

Theorem 4.1. *A real hyper surface of a complex projective space CP^n , $n \geq 2$, under the condition that ξ is a principal curvature vector, is generalized Ricci recurrent.*

Proof. Let us take a real hyper surface of complex projective space CP^n , $n \geq 2$ with generalized recurrent Ricci tensor. Then by virtue of (2.8), it follows from (1.3) that

$$\begin{aligned} g((\nabla_X Q)Y, Z) &= [(2n + 1)\alpha(X) + \beta(X)]g(Y, Z) \\ &\quad + \alpha(X)[hg(AY, Z) - g(A^2Y, Z) - 3\eta(Y)\eta(Z)]. \end{aligned} \tag{4.3}$$

Using (2.9) in (4.3), we get

$$\begin{aligned} & [(2n+1)\alpha(X) + \beta(X)]g(Y, Z) + \alpha(X)[hg(AY, Z) - g(A^2Y, Z) - 3\eta(Y)\eta(Z)] \\ & + 3\eta(Z)g(\phi AX, Y) + 3\eta(Y)g(\phi AX, Z) - (Xh)g(AY, Z) \\ & - hg((\nabla_X A)Y, Z) + g(A(\nabla_X A)Y, Z) + g((\nabla_X A)AY, Z) = 0, \end{aligned} \quad (4.4)$$

for any tangent vectors X, Y , and Z .

Putting $Y = \xi$ and $Z = \phi X$ in (4.4), we get

$$\begin{aligned} & \alpha(X)[hg(A\xi, \phi X) - g(A^2\xi, \phi X)] + 3g(AX, X) \\ & - 3\eta(AX)\eta(X) - (Xh)g(A\xi, \phi X) - hg((\nabla_X A)\xi, \phi X) \\ & + g(A(\nabla_X A)\xi, \phi X) + g((\nabla_X A)A\xi, \phi X) = 0. \end{aligned} \quad (4.5)$$

Let us assume $A\xi = a\xi$. Then by Lemma 2.1, we have a is constant and hence we get

$$(\nabla_X A)\xi = a\phi AX - A\phi AX. \quad (4.6)$$

Using (4.6) in (4.5), we obtain

$$\begin{aligned} & 3g(AX, X) - 3a(\eta(X))^2 - hag(\phi AX, \phi X) \\ & + hg(A\phi AX, \phi X) - g(A\phi AX, A\phi X) + a^2g(\phi AX, \phi X) = 0, \end{aligned} \quad (4.7)$$

for any tangent vector X on M . We choose X as a unit principal curvature vector orthogonal to ξ and by virtue of Lemma 2.2, we have

$$AX = \lambda X \quad \text{and} \quad A\phi X = \bar{\lambda}X,$$

where $\bar{\lambda} = \frac{a\lambda + 2}{2\lambda - a}$. Therefore, we obtain

$$\lambda[\bar{\lambda}^2 - h\bar{\lambda} - (a^2 - ha + 3)] = 0. \quad (4.8)$$

Again from Lemma 2.2, we may write

$$\lambda\bar{\lambda} = \frac{\lambda + \bar{\lambda}}{2}a + 1. \quad (4.9)$$

If $\lambda = \bar{\lambda}$, then (4.9) yields

$$\lambda^2 = a\lambda + 1. \quad (4.10)$$

If 0 occurs as a principal curvature (for a principal vector orthogonal to ξ), then (4.9) implies that all principal curvature must be constant.

We now assuming that 0 is not a principal curvature (again we consider only directions orthogonal to ξ), the relation (4.8) shows that there are at most two distinct principal curvatures. If λ and $\bar{\lambda}$ are distinct, then we have

$$\lambda + \bar{\lambda} = h \quad \text{and} \quad \lambda\bar{\lambda} = -(a^2 - ha + 3),$$

which yields

$$-(a^2 - ha + 3) = \frac{ha}{2} + 1,$$

i.e.,

$$a^2 - \frac{ha}{2} + 4 = 0.$$

Thus, the coefficients in (4.8) are constants and hence so are λ and $\bar{\lambda}$. The final possibility is that all principal curvatures (with principal vectors orthogonal to ξ) satisfy (4.8) and are again constant.

So by Theorem 2.3 and Proposition 4.1, we may conclude the desired result.

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