Transnational Journal of Mathematical Analysis and Applications
Vol. 11, Issue 2, 2023, Pages 103-108
ISSN 2347-9086
Published Online on November 17, 2023

http://jyotiacademicpress.org

FOR MATHEMATICS, IN CONTRADISTINCTION TO ANY EMPIRICAL SCIENCE, THE PREDICATE OF THE CURRENT KNOWLEDGE IN THE SUBJECT SUBSTANTIALLY INCREASES ITS CONSTRUCTIVE AND INFORMAL PART

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Abstract

We assume that the current mathematical knowledge $\mathcal{K}$ is a finite set of statements from both formal and constructive mathematics, which is time-dependent and publicly available. Any theorem of any mathematician from past or present belongs to $\mathcal{K}$. The set $\mathcal{K}$ exists only theoretically. Ignoring $\mathcal{K}$ and its subsets, sets exist formally in ZFC theory although their properties can be time-dependent (when they depend on $\mathcal{K}$) or informal. In every branch of mathematics, the set of all knowable truths is the set of all theorems. This set exists independently of $\mathcal{K}$. Algorithms always terminate. We explain the distinction between algorithms whose existence is provable in ZFC and constructively defined algorithms which are currently known. By using this distinction, we obtain non-trivial statements on decidable sets $\mathcal{X} \subseteq \mathbb{N}$ that

2020 Mathematics Subject Classification: 03A05, 03F65.
Keywords and phrases: constructive algorithms, constructive mathematics, current knowledge in a scientific discipline, current mathematical knowledge, informal mathematics, known algorithms.
Communicated by Francisco Bulnes.
Received October 5, 2023; Revised November 11, 2023

belong to constructive and informal mathematics and refer to the current mathematical knowledge on $\mathcal{X}$. This and the next sentence justify the article title. The current knowledge in any empirical discipline is the whole discipline because truths from the empirical sciences are not necessary truths but working models of truth about particular real phenomena.

We assume that the current mathematical knowledge $\mathcal{K}$ is a finite set of statements from both formal and constructive mathematics, which is time-dependent and publicly available. Any theorem of any mathematician from past or present belongs to $\mathcal{K}$. The set $\mathcal{K}$ exists only theoretically. Ignoring $\mathcal{K}$ and its subsets, sets exist formally in ZFC theory although their properties can be time-dependent (when they depend on $\mathcal{K}$) or informal. In every branch of mathematics, the set of all knowable truths is the set of all theorems. This set exists independently of $\mathcal{K}$. The following true statement:

"*There exists a set* $\mathcal{X} \subseteq \{1,\dots,49\}$ *such that* $card(\mathcal{X}) = 6$ *and* $\mathcal{X}$ *never occurred as the winning six numbers in the Polish Lotto lottery*"

refers to the current non-mathematical knowledge and does not belong to $\mathcal{K}$.

Algorithms always terminate. There is the distinction between existing algorithms (i.e., algorithms whose existence is provable in ZFC) and known algorithms (i.e., algorithms whose definition is constructive and currently known), see [1], [4], [5, p. 9]. By using this distinction, we obtain non-trivial statements on decidable sets $\mathcal{X} \subseteq \mathbb{N}$ that belong to constructive and informal mathematics and refer to the current mathematical knowledge on $\mathcal{X}$, see Statement 1 and [6]-[7]. These results and the next sentence justify the article title. The current knowledge in any empirical discipline is the whole discipline because truths from the empirical sciences are not necessary truths but working models of truth from a particular context, see [8, p. 610].

The feature of mathematics from the article title is not quite new. Observation 1 is known from the beginning of computability theory and shows that the predicate of the current mathematical knowledge slightly increases the informal mathematics.

Observation 1. Church's thesis is based on the fact that the currently known computable functions are recursive, where the notion of a computable function is informal.

In Observation 2, the predicate of the current mathematical knowledge trivially increases the constructive mathematics.

Observation 2. There exists a prime number p greater than the largest known prime number.

In Observations 3-5, the predicate of the current mathematical knowledge trivially increases the formal set theory.

Observation 3. There is no known proof in *ZF* that shows $\emptyset \neq \emptyset$.

Observation 4. There is no known proof in *ZF* that shows the negation of the axiom of determinacy.

Observation 5. There is no known proof in *ZFC* that shows that all cardinal numbers are accessible.

Statement 1. There exists a naturally defined set $\mathcal{C} \subseteq \mathbb{N}$ which satisfies the following conditions (1)-(4):

(1) A known and simple algorithm for every $k \in \mathbb{N}$ decides whether or not $k \in \mathcal{C}$.

(2) There is no known algorithm with no input that returns the logical value of the statement $card(\mathcal{C}) = \omega$.

(3) There is no known algorithm with no input that returns the logical value of the statement $card(\mathbb{N} \setminus \mathcal{C}) = \omega$.

(4) It is conjectured, though so far unproven, that $\mathcal{C}$ is infinite.

Proof. Conditions (1)-(4) hold for

$$\mathcal{C} = \{k \in \mathbb{N} : 2^{2^k} + 1 \text{ is composite}\}.$$

It follows from the following three observations. It is an open problem whether or not there are infinitely many composite numbers of the form $2^{2^k} + 1$, see [3, p. 159]. It is an open problem whether or not there are infinitely many prime numbers of the form $2^{2^k} + 1$, see [3, p. 158]. Most mathematicians believe that $2^{2^k} + 1$ is composite for every integer $k \geqslant 5$, see [2, p. 23]. □

The following 34 phrases or sentences are related to the topic of the article:

(P1) necessary truths in the formal mathematics,

(P2) knowable truths in the formal mathematics,

(P3) provable statements in the formal mathematics,

(P4) Atemporal truth in the formal mathematics,

(P5) current mathematical knowledge $\mathcal{K}$ is a finite set of statements in the public domain,

(P6) known theorems,

(P7) non-mathematical true statements about finite subsets of $\mathbb{N}\setminus\{0\}$ are outside $\mathcal{K}$,

(P8) constructive mathematics,

(P9) temporal truth in the constructive mathematics,

(P10) informal notions,

(P11) informal mathematics,

(P12) computable functions,

(P13) recursive functions,

(P14) Church's thesis,

(P15) computability theory,

(P16) current knowledge in the empirical sciences,

(P17) current knowledge in any empirical discipline is the whole discipline,

(P18) working models of truth about particular real phenomena,

(P19) temporal truth in the empirical sciences,

(P20) consistency of ZF theory,

(P21) algorithms existing in ZFC theory,

(P22) constructively defined algorithms,

(P23) known algorithms,

(P24) sets defined in ZFC theory and their properties that depend on $\mathcal{K}$,

(P25) informal properties of sets defined in ZFC theory,

(P26) mathematical conjectures,

(P27) non-trivial mathematical statements with the predicate of the current mathematical knowledge,

(P28) negation of the axiom of determinacy,

(P29) accessible cardinal numbers,

(P30) known prime numbers,

(P31) prime numbers of the form $2^{2^k} + 1$,

(P32) prime Fermat numbers,

(P33) composite numbers of the form $2^{2^k} + 1$,

(P34) composite Fermat numbers.

An extended version of this article is available at [6].

References

[1] J. Case and M. Ralston, Beyond Rogers' non-constructively computable function, In: The Nature of Computation, Lecture Notes in Computer Science (7921), 45-54, Springer, Heidelberg, 2013.

DOI: https://doi.org/10.1007/978-3-642-39053-1_6

[2] J.-M. De Koninck and F. Luca, Analytic Number Theory: Exploring the Anatomy of Integers, American Mathematical Society, Providence, RI, 2012.

[3] M. Křížek, F. Luca and L. Somer, 17 Lectures on Fermat Numbers: From Number Theory to Geometry, Springer, New York, 2001.

DOI: https://doi.org/10.1007/978-0-387-21850-2

[4] R. Reitzig, How can it be Decidable whether π has Some Sequence of Digits?.

https://cs.stackexchange.com/questions/367/how-can-it-be-decidable-whether-pi-has-some-sequence-of-digits

[5] H. Rogers, Jr., Theory of Recursive Functions and Effective Computability, 2nd Edition, MIT Press, Cambridge, MA, 1987.

[6] A. Tyszka, In mathematics, in contradistinction to any empirical science, the predicate of the current knowledge in the subject substantially increases its constructive and informal part.

https://arxiv.org/abs/1309.2605

[7] A. Tyszka, Statements and open problems on decidable sets $\mathcal{X} \subseteq \mathbb{N}$ that contain informal notions and refer to the current knowledge on $\mathcal{X}$, Creative Mathematics and Informatics 32(2) (2023), 247-253.

https://semnul.com/creative-mathematics/wp-content/uploads/2023/07/creative_2023_32_2_247_253.pdf

[8] B. L. J. Webb, Science, Truth, and Meaning: From Wonder to Understanding, World Scientific, Singapore, 2021.

DOI: https://doi.org/10.1142/12152

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