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VECTOR AUTOREGRESSION AND VECTOR ERROR CORRECTION MODELS: APPLICATION TO ETHIOPIAN AIR LINE DATA

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Abstract

Ethiopian Airline is playing a leading role in transforming Ethiopia into a world class aviation hub of the African continent not only for trade and business but also for the tourism. For the continual of its success, forecasting air travel demand may play a crucial role for an overall effective planning.

The present article has utilized a monthly data from January 2009 up to December 2013 to build vector autoregression (VAR) and VECM models, and also to know the effect of a study target variable Load Factor (LF) to Passenger Revenue (PR), Block Hours (BH), and Distance Flown (DF) at international level.

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The VECM (1) shown that the Load Factor (as a measure of travel demand) is Granger caused by all variables in the short-run except Passenger Revenue and significantly explained by all variables in the long-run.

1. Introduction

Ethiopian Airline (Ethiopian) is the flag carrier of Ethiopia. It is founded in December 21, 1945 and became operational in April 08, 1946. It is under complete government ownership. During the past sixty five plus years, Ethiopian has become one of the continent's leading carriers, unrivalled in Africa for efficiency and operational success, turning profits for almost all the years of its existence. Accordingly, air travel demand can be considered as the customers (air passengers') degree of response (utilization) to the total transportation services provided by an airline.

A review of the available literatures on air travel demand reveal that the building of models to estimate demand for air-passengers can take many forms, each depending on the objective of the model being developed. Ippotito [10] used a cross-sectional model to estimate the origin and destination's demand for airlines (US domestic) at each end of the route that incorporated a measure of service. The results of his study confirmed the long held belief that demand is sensitive to flight frequency and availability of "excess" seats, and that the quantity of seats offered is positively and significantly affected by regulated price. It also confirmed that the price elasticity of demand increases with flight distance.

Kumar and Stephanedes [16] studied the impact of air travel supply on the demand and vice-versa between Twin Cities and Chicago. They used a time series analysis as a tool for estimating the impact of air travel supply on the demand and vice-versa on non-stop air routes. Another study by Ghobrial [6] presents an econometric model that estimates the aggregate demand for an airline. The demand is expressed in terms of airline network structure, operating characteristics and firm-specific variables. Poore [25] has conducted a study to test the hypothesis that forecasts of the future demand for air transportation offered by airplane manufacturers and aviation regulators are reasonable and representative of the trends implicit in actual experience. The test compared forecasts issued by Boeing, McDonnell Douglas, Airbus Industry and the International Civil Aviation Organization with actual experience and the results of a baseline model of the demand for revenue passenger kilometers (RPKs). The model is the combination of two equations describing the RPKs demanded by the high- and the low income groups, respectively. Seraj et al. [26] developed several models for the air travel demand with different combinations of fourteen explanatory variables utilizing stepwise regression technique. Among all candidate models, the model obtained by using least square technique i-th the two variables, (i.e., total expenditures and population size) for international air travel demand models in Saudi Arabia was the most appropriate model to represent the demand for international air travel in Saudi Arabia.

Kulendran and Witt [15] generated one, four and six quarter ahead forecasts of international business passengers to Australia from the following four countries: Japan, New Zealand, the United Kingdom and the United States. They considered various forecasting models: the error correction model (ECM), the structural time series model (STSM), the basic structural model (BSM), and ARIMA model. They concluded that forecasting performance varies with the forecasting horizon and depends on the adequate detection of seasonal unit roots. Consequently, ARIMA and BSM models are the most accurate for short term forecasting (one-quarter ahead) whereas ECM outperforms for medium term forecasting (four and six quarters ahead).

Another study for air travel demand forecasting is done by Grosche et al. [8]. According to their research, there are some variables that can affect the air travel demand, including population, GDP and buying power index. They considered GDP as a representative variable for the level of economic activity.

Tsekeri [28] has also estimated the short and long-term response of air passengers to change in relative air-sea travel cost components in competitive markets using a dynamic demand model. The model demonstrated the importance of considering the past volumes of air passengers and relative travel cost components to explain current air travel demand.

Constantinos [2] examined whether or not combining forecasts from autoregressive-integrated-moving average (ARIMA) and seasonal autoregressive-integrated-moving average (SARIMA) models helps to improve the forecasting accuracy of Canadian air transportation sector in domestic, transborder (US) and international flights. His study also provided forecasts of air passengers in Canada based on various time series forecasting models.

This article utilized a monthly data from January 2009 up to December 2013 to build vector autoregression (VAR) model. Each series are found to be integrated of order one ($I(1)$). The three information criteria AIC, SC, and HQ recommended one lag length. Johansen cointegration test indicated only one long-term equilibrium relationship occurred between the variables. This immediately implied the legitimacy of vector error correction (VEC) model of order one to be fitted than a pure VAR (1) model for the time series data. As one footstep before out-of-sample forecasting, the VECM (1) model has been checked for its accuracy with the aid of RMSE, MAE, MAPE, and Theil-U statistics. The summary result of VECM (1) shown that Load Factor (as a measure of travel demand) is Granger caused by all variables in the short-run except Passenger Revenue and significantly explained by all variables in the long-run. At last, forecasting is made for Ethiopian International air travel demand (Load Factor).

2. Data and Statistical Methodology

2.1. Data source

This study considered a monthly Ethiopian airline's data for international flights over the time period January 2009 – December 2013 which have been obtained from the Head Quarter Office of Ethiopian Airline located in Bole sub-city, Addis Ababa, Ethiopia.

2.2. Definition and variables of the study

The incorporated variables are somewhat technical and need a brief description as follows.

(1) International load factor (LF): is the target variable of the study, which describes the percentage of actual air seats purchased out of the total (available) seats, provided per month by an airline for international flight. Mathematically, it is expressed as

$$\frac{\text{Number of Passengers}}{\text{Aircraft Configured Seat Number}}.$$

(2) International passenger revenue (PR): is the monthly aggregate revenue that would be earned from each individual international flight in millions of USD.

(3) International block hours (BH): describes monthly flight duration and is a summation of each individual international flight's time difference between engine on and engine off. It is measured by thousands of hours.

(4) International distance flown (DF): is a monthly distance covered by all international flights. It is cumulative kilometers flown by each international flight. The study measures this variable in millions of kilometers.

2.3. Statistical methodology

This section deals with the Vector Autoregressive (VAR) models for stationary and cointegrated variable(s). We have also discussed model specification and parameter estimation. In other section, we discuss with Structural Vector Autoregressive (SVAR) Analysis (i.e., Granger Causality, Impulse Response Functions (IRF), and Forecast Error Variance Decomposition (FEVD)).

2.3.1. Vector autoregressive (VAR) models

We have applied Vector Autoregression (VAR) model, proposed by Sims [27]. Forecasts from VAR models are quite flexible because they can be made conditional on the potential future paths of specified variables in the model.

2.3.2. Integration ($I(d)$)

If a series is stationary without any differencing it is designated as $I(0)$. A series that has stationary first difference is designated $I(1)$. In general if a non-stationary time series has to be differenced d times to make it stationary, that time series is said to be integrated of order d and denoted as $I(d)$ (Gujarati [9]; Pole et al. [24]; Weigend and Gershenfeld [29]).

2.3.3. Stationary vector autoregressive model

Let $Y_t = (Y_{1t}, Y_{2t}, \dots, Y_{nt})'$ denote a $(n \times 1)$ vector of time series variables. A VAR model with p lags can then be expressed as follows:

$$Y_t = c + \prod_1 Y_{t-1} + \prod_2 Y_{t-2} + \cdots + \prod_p Y_{t-p} + \varepsilon_t, \quad t = 1, \dots, T, \quad (2.1)$$

where c denotes an $(n \times 1)$ vector of constants and $\prod_i$, for $i = 1, 2, \dots, p$, is an $(n \times n)$ coefficient matrix of autoregressive coefficients. ε_t is an $(n \times 1)$ unobservable zero mean white noise vector process (serially uncorrelated) with time invariant covariance matrix $\sum$, i.e.,

$$E(\varepsilon_t) = 0 \text{ and } \operatorname{Cov}(\varepsilon_t, \varepsilon_s) = E(\varepsilon_t, \varepsilon_s') = \begin{cases} \sum, & \forall t = S, \\ 0, & \forall t \neq S, \end{cases} \quad (2.2)$$

with $\sum$ an $(n \times n)$ symmetric positive definite matrix.

Let c_i denote the i-th element of the vector c and let $\prod_{ij}^{(1)}$ denote the element on i-th row, j-th column of the matrix $\prod_1$. Then the first row of the vector system in (2.1) specifies that

$$Y_{1t} = c_1 + \prod_{11}^{(1)} Y_{1,t-1} + \prod_{12}^{(1)} Y_{2,t-1} + \cdots + \prod_{1n}^{(1)} Y_{n,t-1} + \prod_{11}^{(2)} Y_{1,t-2} + \prod_{12}^{(2)} Y_{2,t-2}$$

$$+ \cdots + \prod_{1n}^{(2)} Y_{n,t-2} + \cdots + \prod_{11}^{(p)} Y_{1,t-p} + \prod_{12}^{(p)} Y_{1,t-p} + \cdots + \prod_{1n}^{(p)} Y_{n,t-p} + \varepsilon_{1t}, \quad (2.3)$$

$Y_{2t}, \ldots, Y_{nt}$ can also be written in the same manner as Y_{1t}.

In lag operator notation, the VAR(p) is written as

$$\prod(L)Y_t = c + \varepsilon_t, \tag{2.4}$$

where $\prod(L) = I_n - \prod_1 L - \cdots - \prod_p L_p$. The VAR($p$) is stable if the roots of

$$\det\left[\prod(L)\right] = \det\left(I_n - \prod_1 L - \cdots - \prod_p L_p\right) = 0 \tag{2.5}$$

lie outside the complex unit circle (have modulus greater than one), or, equivalently, if the eigenvalues of the companion matrix

$$F = \begin{pmatrix} \prod_1 & \prod_2 & \cdots & \prod_n \\ I_n & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & I_n & 0 \end{pmatrix}$$

have modulus less than one. Assuming that the process has been initialized in the infinite past, then a stable VAR(p) process is stationary and *ergodic* (i.e., if sample mean, sample autocovariance, and sample autocorrelation converge in probability to their respective population moments) with time invariant means, variances, and autocovariances.

If Y_t in (2.1) is covariance stationary, then the unconditional mean is given by

$$\mu = \left(I_n - \prod_1 - \cdots - \prod_p\right)^{-1} c. \tag{2.6}$$

The mean-adjusted form of the VAR(p) is then

$$Y_t - \mu = \prod_1(Y_{t-1} - \mu) + \prod_2(Y_{t-2} - \mu) + \cdots + \prod_p(Y_{t-p} - \mu) + \varepsilon_t. \tag{2.7}$$

The basic VAR(p) model may be too restrictive to represent sufficiently the main characteristics of the data. In particular, other deterministic terms (for instance, a linear time trend) and stochastic exogeneous variables may be required to represent the data properly. The general form of the VAR(p) model with deterministic terms and exogeneous variables is given by

$$Y_t = \prod_1 Y_{t-1} + \prod_2 Y_{t-2} + \cdots + \prod_p Y_{t-p} + \Phi D_t + GX_t + \varepsilon_t, \tag{2.8}$$

where D_t represents an $(l \times 1)$ matrix of deterministic components, X_t represents an $(m \times 1)$ matrix of exogeneous variables, and Φ and G are parameter matrices.

2.3.4. Testing stationarity

To test stationarity of the series we have applied Augmented Dickey-Fuller (ADF) test due to Dickey and Fuller [3, 4], and the Phillip-Perron (PP) test due to Phillips [22] and Phillips and Perron [23].

2.3.5. Specification of VAR order

The general approach is to fit VAR models with orders $m = 0, \ldots,$ $p_{\max}$ and choose the value of m which minimizes some model selection criteria (Lütkepohl [17]).

The general model selection criteria have the form:

$$IC(p) = \ln\left|\overline{\sum}(p)\right| + c_T \cdot \varphi(n,\ p), \tag{2.9}$$

where $\overline{\sum}(p) = T^{-1}\sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}'_t$ is the residual covariance matrix without a degrees of freedom correction from a VAR(p) model, c_T is a sequence indexed by the sample size T, and $\varphi(n,\ p)$ is a penalty function which penalizes large VAR(p) models. In this paper, we considered Akaike information criterion (AIC), Schwartz (SC) information criterion, and Hannan-Quinn (HQ) information criterion to pick out an optimal lag order for the VAR t:

$$AIC(p) = \ln\left|\overline{\sum}(p)\right| + \frac{2}{T} pn^2, \tag{2.10}$$

$$SC(p) = \ln\left|\sum(p)\right| + \frac{\ln T}{T} pn^2, \tag{2.11}$$

$$HQ(p) = \ln\left|\sum(p)\right| + \frac{2\ln(\ln T)}{T} pn^2. \tag{2.12}$$

The AIC criterion asymptotically overestimates the order with positive probability, whereas the SC and HQ criteria estimate the order consistently under fairly general conditions if the true order p is less than or equal to $p_{\max}$.

2.3.6. Cointegration analysis

Engle and Granger [5] developed the theory that there exists the special case where linear combinations of nonstationary processes are stationary. Consider two $I(1)$ processes, X_{1t} and X_{2t}, if there exists a linear combination of the two processes the two $I(1)$ processes are considered to be $CI(1, 1)$. Broadly, contegrating relationships can be either single or multiple as follows.

2.3.6.1. Single cointegration relationship

Let $Y_t = (Y_{1t}, Y_{2t}, \ldots, Y_{nt})'$ denote an $(n \times 1)$ vector of $I(1)$ time series. Y_t is said to be cointegrated if there exists an $(n \times 1)$ vector $\beta = (\beta_1, \beta_2, \ldots, \beta_n)'$ such that

$$\beta' Y_t = \beta_1 Y_{1t} + \beta_2 Y_{2t} + \cdots + \beta_n Y_{nt} \sim I(0). \tag{2.13}$$

In words, the non-stationary time series in Y_t are cointegrated if there is a linear combination that is stationary or $I(0)$. If some elements of β are equal to zero then only the subset of the time series in Y_t with non-zero coefficients is cointegrated.

Normalization

The cointegration vector β in (2.13) is not unique since for any scalar c the linear combination $c\beta' Y_t = \beta^{*'} Y_t \sim I(0)$. Hence, some normalization assumption is required to uniquely identify β. A typical normalization is

$$\beta = (1, -\beta_2, \ldots, -\beta_n)'.$$

So that the cointegration relationship may be expressed as

$$\beta' Y_t = Y_{1t} - \beta_2 Y_{2t} - \cdots - \beta_n Y_{nt} \sim I(0),$$

or

$$Y_{1t} = \beta_2 Y_{2t} + \cdots + \beta_n Y_{nt} + U_t, \tag{2.14}$$

where $U_t \sim I(0)$. In (2.14), the error term U_t is often referred to as the *disequilibrium error* or the *cointegrating residual*. In long-run equilibrium, the disequilibrium error U_t is zero and the long-run equilibrium relationship is

$$Y_{1t} = \beta_2 Y_{2t} + \cdots + \beta_n Y_{nt}. \tag{2.15}$$

2.3.6.2. Multiple cointegration relationships

If the $(n \times 1)$ vector Y_t is cointegrated, there may be r, $0 < r < n$, linearly independent cointegrating vectors. For example, let $n = 3$ and suppose there are $r = 2$ cointegrating vectors $\beta_1 = (\beta_{11}, \beta_{12}, \beta_{13})'$ and $\beta_2 = (\beta_{21}, \beta_{22}, \beta_{23})'$. Then $\beta_1' Y_t = \beta_{11} Y_{1t} + \beta_{12} Y_{2t} + \beta_{13} Y_{3t} \sim I(0)$, $\beta_2' Y_t = \beta_{21} Y_{1t} + \beta_{22} Y_{2t} + \beta_{23} Y_{3t} \sim I(0)$ and the (2×3) matrix

$$B' = \begin{pmatrix} \beta_1' \\ \beta_2' \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \end{pmatrix} \tag{2.16}$$

forms a basis for the space of cointegrating vectors. The linearly independent vectors β_1 and β_2 in the cointegrating basis B are not unique unless some normalization assumptions are made. Furthermore, any linear combination of β_1 and β_1, e.g., $\beta_3 = c_1\beta_1 + c_2\beta_2$, where c_1 and c_1 are constants, is also a cointegrating vector.

2.3.6.3. Testing for cointegration using Johansen's methodology

We applied the Johansson's procedure [12]. The procedure begins with unrestricted VAR involving potentially non-stationary variables.

The starting point of Johansen's procedure [11, 12] in determining the number of cointegrating vectors is the VAR representation of Y_t. It is assumed a vector autoregressive model of order p and is expressed as follows in (2.17).

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + BX_t + \varepsilon_t, \tag{2.17}$$

where Y_t is an n-vector of non-stationary $I(1)$ variables (i.e., the non-stationary series variables in Y_t are differenced once to achieve stationarity, then Y_t is said to be integrated of order one). This would be then written as $Y_t \sim I(1)$), X_t is a d-vector of deterministic (other exogeneous) variables, and ε_t is a vector of innovations.

(2.17) can be re-written as:

$$\Delta Y_t = \prod Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + BX_t + \varepsilon_t, \tag{2.18}$$

where

$$\prod = \sum_{i=1}^{p} A_i - I, \; \Gamma_i = -\sum_{j=i+1}^{p} A_j. \tag{2.19}$$

Granger's representation theorem asserts that if the coefficient matrix $\prod$ has reduced rank $r < n$, then there exist $n \times r$ matrices α and β each with rank r such that $\prod = \alpha\beta'$ and $\beta' Y_t$ is $I(0)$, where r is the number of cointegrating relations (the *cointegrating rank*) and each column of it represent the cointegrating vector. The elements of α are known as the *adjustment parameters* in the VEC model. It can be shown that for a given r, the maximum likelihood estimator of β defines the combination of y_{t-1} that yields the r largest canonical correlations of ΔY_t with Y_{t-1} after correcting for lagged differences and deterministic variables when present.

Johansen [11] proposed two tests for estimating the number of cointegrating vectors: the trace statistic and maximum eigenvalue statistic tests.

Then the trace statistic is computed as

$$\hat{\lambda}_{trace}(r) = -T\sum_{i=r+1}^{n}\log[1-\hat{\lambda}_i]. \tag{2.20}$$

The maximum eigenvalue statistic tests the null hypothesis of r cointegrating relations against the alternative of $r+1$ cointegrating relations for $r = 0, 1, 2, \ldots, n-1.$ This test statistic is computed as

$$\hat{\lambda}_{\max}(r, r+1) = -T\log[1-\hat{\lambda}_{r+1}], \tag{2.21}$$

where $\hat{\lambda}_{r+1}$ is the $(r+1)$-th ordered eigenvalue of $\prod$, and T is the sample size. The critical values tabulated by Johansen and Juselius [14] will be used for these tests.

2.4. Vector error correction (VEC) models

The following equation

$$\Delta Y_t = \prod Y_{t-1} + \sum_{i=1}^{p-1}\Gamma_i \Delta Y_{t-i} + BX_t + \varepsilon_t$$

is known as a *vector error correction model* (VECM), where $\prod = -I_n + \sum_{i=1}^{p} A_i$, I_n is the identity matrix, and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$.

The above specification of VECM contains information on both the short and the long-run adjustment to changes in Y_t via estimating Γ and $\prod$, respectively. Matrix $\prod$ can be decomposed as $\prod = \alpha\beta'$, where α is $n \times r$ matrix of speed of adjustments towards the long-run equilibrium, and β is an $n \times r$ matrix of parameters which determines the cointegrating relationships of long-run coefficients such that $\beta' Y_{t-n}$ represent the multiple cointegration relationships. The columns of β are interpreted as long-run equilibrium relationships between variables.

Values of α close to zero imply slow convergences and $r, 0 \leq r \leq n,$ is the rank of the matrix $\prod$ and represents the number of cointegrating vectors in the system which can be determined using the Johansen Maximum Likelihood method.

2.5. Model checking

It is an obligatory activity to investigate validity and reliability of all inference procedures made by VARs and VECMs before one is going to use these models to forecast future patterns of series. There are several tests for checking forecasting capability (adequacy) of these models.

2.5.1. Test of residual autocorrelation

We applied two most popular tests for autocorrelation of residuals, i.e., Breusch-Godfrey LM tests and Portmanteau tests.

2.5.2. Normality of the residuals

We checked the normality of residuals from VAR and VECM, using Jarque and Bera's test [1].

2.5.3. Forecasting

The ultimate goal of estimating VAR and VECM is forecasting. Consider first the problem of forecasting future values of Y_T when the parameters $\prod$ of the VAR(p) process are assumed to be known and there are no deterministic terms or exogeneous variables. The best linear predictors, in terms of minimum mean squared error (MSE), of Y_{T+1} or 1-step forecast based on information available at time T is

$$Y_{T+1|T} = c + \prod\nolimits_1 Y_T + \cdots + \prod\nolimits_p Y_{T-p+1}, \tag{2.22}$$

for $T \geq p$.

Forecasts for longer horizons h (h-step forecasts) can be obtained using the chain-rule of forecasting as

$$Y_{T+h|T} = c + \prod_1 Y_{T+h-1|T} + \cdots + \prod_p Y_{T+h-p|T}, \tag{2.23}$$

where $Y_{T+j|T} = Y_{T+j}$ for $j \leq 0$. The h-step forecast errors may be expressed as

$$Y_{T+h} - Y_{T+h|T} = \sum_{s=0}^{h-1} \Psi_s \varepsilon_{T+h-s}, \tag{2.24}$$

where the matrices Ψ_s are determined by recursive substitution,

$$\Psi_s = \sum_{j=1}^{p-1} \Psi_{s-j} \prod_j, \tag{2.25}$$

with $\Psi_0 = I_n$ and $\prod_j = 0_{n \times n}$ for $j > p$. The forecasts are unbiased since all of the forecast errors have expectation zero, and the MSE matrix for $Y_{T+h|T}$ is

$$\sum(h) = MSE\left(Y_{T+h} - Y_{T+h|T}\right)$$

$$\sum(h) = MSE\left(\sum_{s=0}^{h-1} \Psi_s \varepsilon_{T+h-s}\right)$$

$$= \sum_{s=0}^{h-1} \Psi_s \sum \Psi_s'. \tag{2.26}$$

Now consider forecasting Y_{T+h} when the parameters of the VAR(p) process are estimated using multivariate least squares. The best linear predictor of Y_{T+h} is now

$$\hat{Y}_{T+h|T} = \widehat{\prod}_1 \hat{Y}_{T+h-1|T} + \cdots + \widehat{\prod}_p \hat{Y}_{T+h-p|T}, \tag{2.27}$$

where $\widehat{\prod}_j$ are the estimated parameter matrices. The h-step forecast error is given by

$$Y_{T+h} - \hat{Y}_{T+h|T} = \sum_{s=0}^{h-1} \Psi_s \varepsilon_{T+h-s} + (Y_{T+h|T} - \hat{Y}_{T+h|T}), \tag{2.28}$$

and the term $Y_{T+h|T} - \widehat{Y}_{T+h|T}$ captures the part of the forecast error due to estimating the parameters of the VAR. The MSE matrix of the h-step forecast is then,

$$\widehat{\sum}(h) = \sum(h) + MSE\left(Y_{T+h|T} - \widehat{Y}_{T+h|T}\right). \tag{2.29}$$

In practice, the second term $MSE\left(Y_{T+h|T} - \widehat{Y}_{T+h|T}\right)$ is often ignored and $\widehat{\sum}(h)$ is computed using (2.27) as:

$$\widehat{\sum}(h) = \sum_{s=0}^{h-1} \widehat{\Psi}_s \widehat{\sum} \widehat{\Psi}'_s, \tag{2.31}$$

with $\widehat{\Psi}_s = \sum_{j=1}^{s} \widehat{\Psi}_{s-j} \widehat{\prod}_j$. Lütkepohl [17] gave an approximation to $MSE\left(Y_{T+h|T} - \widehat{Y}_{T+h|T}\right)$ which may be interpreted as a finite sample correction to (2.31).

2.5.4. Measures of forecasting accuracy

We evaluated the forecasting performance of a VEC model, ranging from mean error (ME) measures to Theil's U statistic measure.

2.6. Structural vector autoregressive (SVAR) analysis

We will interpret a VAR model in the following ways.

2.6.1. Granger causality test

Granger [7] has defined a concept of causality that if a variable x affects a variable z, the former should help improving the predictions of the latter variable.

For instance, in a bivariate VAR(p) model for $Y_t = (Y_{1t}, Y_{2t})'$, Y_2 fails to Granger-cause Y_1 if all of the p VAR coefficient matrices $\prod_1, \ldots, \prod_p$ are lower triangular. That is, the VAR (p) model has the form

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \prod_{11}^{(1)} & 0 \\ \prod_{21}^{(1)} & \prod_{22}^{(1)} \end{pmatrix} \begin{pmatrix} Y_{1,t-1} \\ Y_{2,t-1} \end{pmatrix} + \cdots + \begin{pmatrix} \prod_{11}^{(p)} & 0 \\ \prod_{21}^{(p)} & \prod_{22}^{(p)} \end{pmatrix} \begin{pmatrix} Y_{1,t-p} \\ Y_{2,t-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}. \quad (2.32)$$

So that all of the coefficients on lagged values of Y_2 is zero in (2.32) for Y_1. Similarly, Y_1 fails to Granger-cause Y_2 if all of the coefficients on lagged values of Y_1 are zero in the equation for Y_2.

2.6.2. Impulse response functions

Impulse response function is an important tool in a VAR system in revealing the direction and magnitude at which one variable (especially the target variable) reacts to the change (shock) applied on the other exogeneous variables in the system.

3. Statistical Results and Discussions

3.1. Descriptive analysis and time plot

Table 3.1 exhibits the mean, minimum, maximum, standard deviation, coefficient of variation (CV), Jarque-Bera statistics, and the corresponding probability values (P-values). From Table 3.1, the Load Factor and Passenger Revenue are the lowly and highly scattered series in the study with a CV of 5.312% and 26.383%, respectively. Block hours and Distance flown series are almost equally dispersed. Regarding normality, all the P-values of the Jarque-Bera statistic in the Table 3.1 are greater than 5% suggesting that initially all the series are normally distributed.

Table 3.1. Summary of descriptive statistics for all original series

Series	Mean	Minimum	Maximum	Std. Dev.	CV	Jarque-Bera	Probability
Load factor	71.19722	64.35880	78.9056	3.782256	0.05312	1.139294	0.565725*
Block hours	14.20452	7.995810	19.3971	3.031984	0.21350	2.527538	0.282587*
Distance flown	9.667997	5.398781	13.4377	2.125080	0.21981	2.469979	0.290838*
Passenger revenue	77.494972	41.566976	122.0000	20.445475	0.26383	2.907045	0.233746*

***P-values < 0.05: Statistically significant**

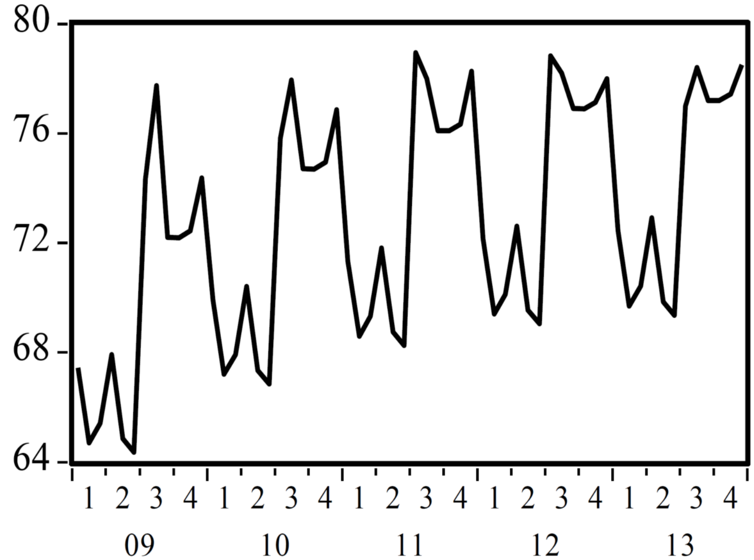
On its part, Table 3.2 below presents the correlation matrix for the four series and all are significantly inter-correlated as the figures tend to one in magnitude. Accordingly, Load factor of the aviation is highly correlated with Passenger revenue, Block hours, and Distance flown.

Table 3.2. Correlation matrix of the variables

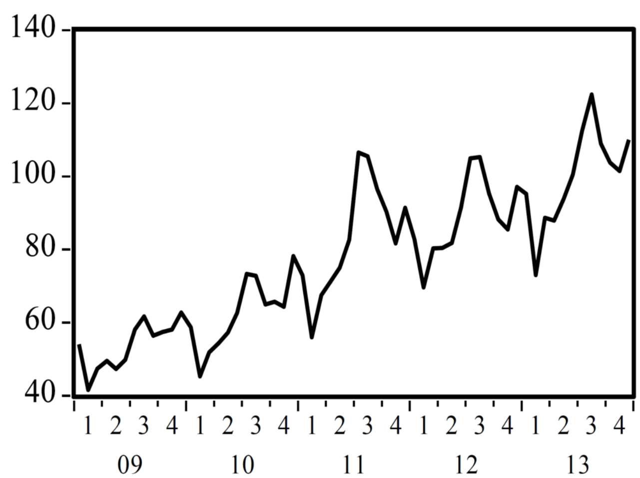
Variables	Load factor	Block hours	Distance flown	Passenger revenue
Load factor	1.000000	0.876008	0.873992	0.899532
Block hours	0.876008	1.000000	0.999878	0.991071
Distance flown	0.873992	0.999878	1.000000	0.990727
Passenger revenue	0.899532	0.991071	0.990727	1.000000

From Figure 3.1, it can be seen that there is a clear seasonality and a general upward trend in all the series. This implies that all the data are not stationary. But it should be strongly noticed that only graph inspections are not enough to conclude the series are seasonal and patterned. There are standard tests for both seasonality and stationarity which has been discussed previously in methodology and will be applied in analysis of the data as follow.

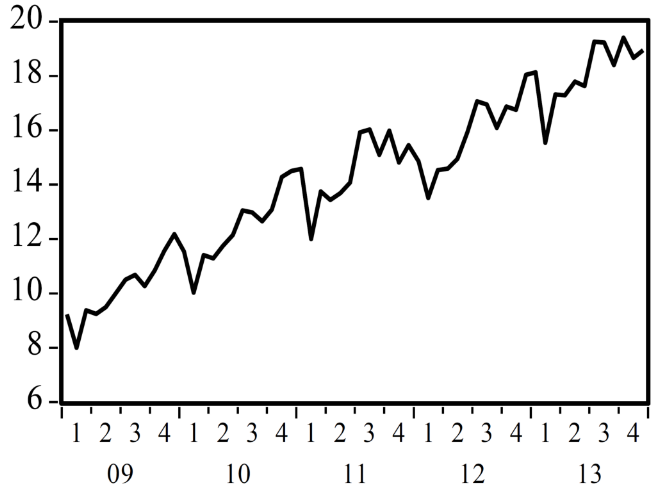
Plot of Load Factor (LF) in % versus Time
Y-Axis : LF and X-Axis : Time



Plot of Passenger Revenue (PR) in Millions of USD versus Time
Y-Axis : PR and X-Axis : Time



Plot of Block Hours (BH) in Thousands of Hours versus Time
Y-Axis : BH and X-Axis : Time



Plot of Distance Flown (DF) in Millions of Kilometers versus Time
Y-Axis : DF and X-Axis : Time

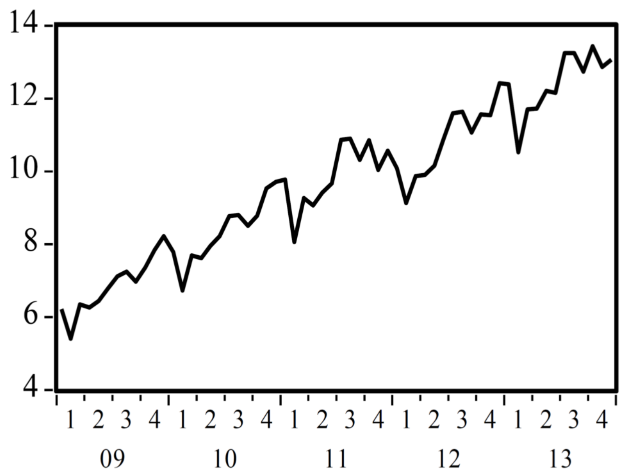


Figure 3.1. Time plots of each original series.

3.2. Seasonality test

When using X-12 ARIMA for seasonal adjustment, two diagnostics commonly used to determine seasonality are M7, a diagnostic developed at Statistics Canada for X-11-ARIMA, and the F-tests for seasonality of the series and residuals. Then the variable(s) in which seasonality is observed will be seasonally adjusted.

Table 3.3 presents the full F-tests for seasonality of the original Load Factor (LF). The *combined test for the presence of identifiable seasonality* indicates that LF series has a seasonal pattern that can be identified by X-12 ARIMA. The M7 diagnostic ($0.390 < 1$) also strengthens the identifiability of the LF series with the aid of X-12 ARIMA. From the Table 3.3, the F-tests assert that seasonality exists in the monthly original LF series at 0.1% level of significance. But the good is that, before seasonal adjustment, the seasonality never passed to the years with a confidence of 95% and also the residuals of the series are free from seasonality at 1% significance level. In addition to M7, all the M-statistics are shown to be less than one, and hence the Q-statistic (0.45) produced from them is also less than one. This condition assures that the seasonal adjustment performed on LF is acceptable.

Table 3.3. F-tests for seasonality and adjustment quality diagnostics of original LF

Test for the presence of seasonality assuming stability:				
	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	1514.9564	11	137.72331	181.173**
Residuals	36.4885	48	0.76018	
Total	1551.4449	59		
****Seasonality present at the 0.1 percent level**				
Nonparametric test for the presence of seasonality assuming stability:				
	Kruskal-Wallis statistic	Degrees of freedom	Probability level	
	56.7443	11	0.000%	
Seasonality present at the one percent level				
Moving seasonality test:				
	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	2.7531	4	0.688283	0.898
Error	33.7353	44	0.766712	
No evidence of moving seasonality at the five percent level				
COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY:				
IDENTIFIABLE SEASONALITY PRESENT				
Test for the presence of residual seasonality:				
No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.01				
No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 1.64				
No evidence of residual seasonality in the last 3 years at the 5 percent level.				
M1 = 0.532, M2 = 0.429, M3 = 0.856, M4 = 0.563, M5 = 0.433, M6 = 0.098, M7 = 0.390, Q = 0.45				

The remaining pre adjustment tests for Passenger revenue (PR), Block hours (BH), and Distance flown (DF) can be seen from Table A1 (a)-(c) in Appendix.

3.3. Post-seasonal adjustment features

From Table 3.4 below, at 1% level of risk, the *test for residual seasonality* (at 1% significance level) shows that there is no any estimable seasonal effect left in the seasonally adjusted series of LF and its irregular component as it is also indicated by the F-tests at 0.1% and 1% (for *Kruskal-Wallis test*) significance level. The *combined test for the presence of seasonality* together with M7 diagnostic (a value of 3 which is greater than 1) is also assuring that no more seasonal adjustment will be necessary at 1% significance level. Similar deductions can be drawn from the *tests of residual seasonality* (with 99% confidence) and F-tests (with 99.9% confidence) in Tables A2 (a)-(c) in Appendix which exhibit that seasonality is completely removed from PR, BH, and DF series. Moreover, the *combined tests for seasonality* and M7 (greater than 1 for each variable) guarantee that no more seasonal adjustment is required for each variable.

Table 3.4. F-tests for seasonality of load factor series after adjustment

Test for the presence of seasonality assuming stability:				
	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	0.0234	11	0.00213	0.004
Residuals	26.5476	48	0.55307	
Total	26.5710	59		
No evidence of stable seasonality at the 0.1 percent level				
Nonparametric test for the presence of seasonality assuming stability:				
	Kruskal-Wallis statistic	Degrees of freedom	Probability level	
	1.7659	11	99.916%	
No evidence of seasonality at the one percent level				
Moving seasonality test:				
	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	0.7577	**4**	0.189416	1.214
Error	6.8632	44	0.155982	
No evidence of moving seasonality at the five percent level				
COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY:				
IDENTIFIABLE SEASONALITY NOT PRESENT				
Test for the presence of residual seasonality:				
No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.02				
No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 1.38				
No evidence of residual seasonality in the last 3 years at the 5 percent level.				
M7 = 3.000				

3.4. Stationarity test for individual series

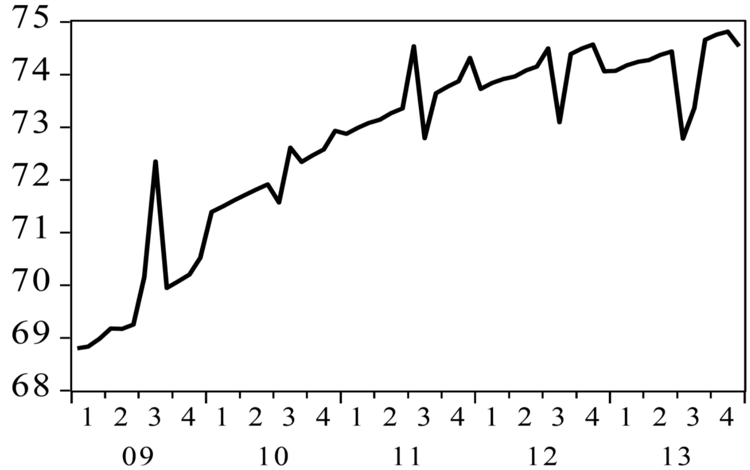
In practice, using the non-stationary time series in VAR modelling is problematic with regard to statistical inference since the standard statistical tests used for inference are based on the condition that all of the series used must be stationary.

Thus, inspections and standard testes should be conducted on each variable for the presence of unit root(s) and in so doing the order of integration of each series is determined.

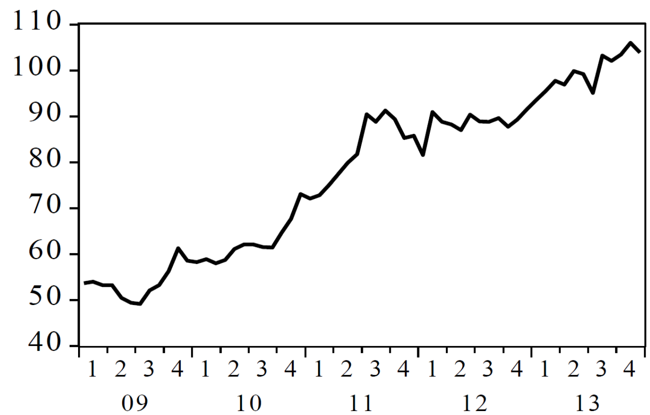
3.4.1. Visual inspection

As shown in Figure 3.2, all the series are seasonally adjusted but they are still with an increasing pattern in line with time increment. This means that all the series are non-stationary. But the time plots should not be the only instruments to detect stationarity of the series. Rather the clue obtained from the time plots ought to be authenticated by standard tests for stationarity (unit root tests).

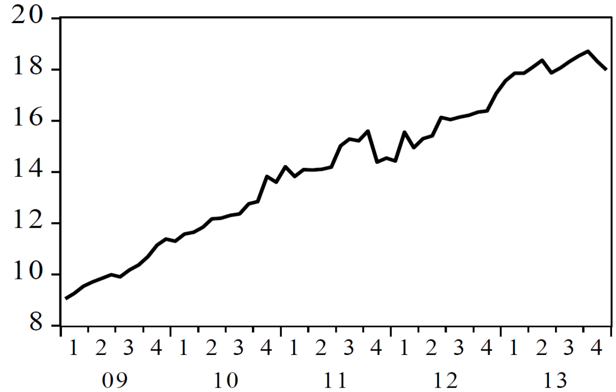
Plot of Seasonally Adjusted LF in % versus Time
Y-Axis : Seasonally Adjusted LF and X-Axis : Time



Plot of Seasonally Adjusted PR in Millions of USD versus Time
Y-Axis : Seasonally Adjusted PR and X-Axis : Time



Plot of Seasonally Adjusted BH in Thosands of Hours versus Time
Y-Axis : Seasonally Adjusted BH and X-Axis : Time



Plot of Seasonally Adjusted DF in Millions of Kilometers versus Time
Y-Axis : Seasonally Adjusted DF and X-Axis : Time

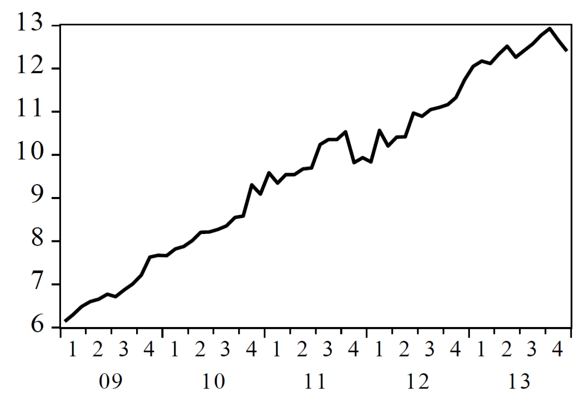


Figure 3.2. Post-seasonal adjustment time plots for each series.

3.4.2. Unit root test

Unit root tests are confirmatory strive for stationarity detection. Augmented Dickey-Fuller test and a Phillips and Perron test are employed to test stationarity and determine the maximum order of integration of each series. The hypothesis of these tests will be as follows.

H_0: The series is non-stationary.

H_1: The series is stationary.

Tables 3.5 and 3.6 below provide the outcome of ADF and PP tests. The critical values used for the tests are the McKinnon [18] critical values.

Table 3.5. Unit root test results (at level)

Series	With intercept				With intercept and trend			
	Test Statistic		Prob.*		Test statistic		Prob.*	
	ADF	PP	ADF	PP	ADF	PP	ADF	PP
LF	– 1.44	– 2.16	0.56	0.22	– 2.93	– 3.33	0.16	0.07
PR	1.34	1.26	0.99	0.99	– 1.72	– 1.91	0.74	0.65
BH	1.56	1.81	0.99	0.99	– 2.19	– 2.04	0.49	0.57
DF	1.94	2.11	0.99	0.99	– 2.11	– 1.93	0.53	0.64
Critical value (5%)	– 2.88				– 3.44			

*MacKinnon [19] one-sided p-values.

Once it is confirmed that all the series are non-stationary, the next step is to go for differencing so as to make the data stationary.

Table 3.6. Unit root test results (after first difference)

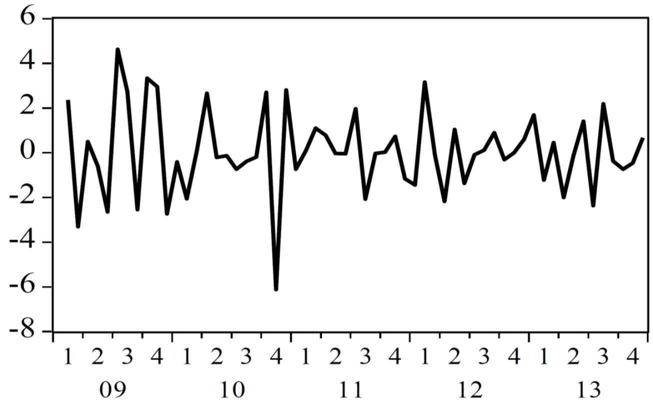
Series	With intercept				With intercept and trend			
	Test statistic		Prob.*		Test statistic		Prob. *	
	ADF	PP	ADF	PP	ADF	PP	ADF	PP
LF	– 13.07	– 19.33	0.00	0.00	– 13.03	– 19.26	0.00	0.00
PR	– 15.08	– 15.01	0.00	0.00	– 15.29	– 15.31	0.00	0.00
BH	– 15.12	– 15.29	0.00	0.00	– 15.45	– 16.54	0.00	0.00
DF	– 15.47	– 15.42	0.00	0.00	– 15.94	– 16.83	0.00	0.00
Critical Value (5%)	– 2.88				– 3.44			

*MacKinnon [19] one-sided p-values

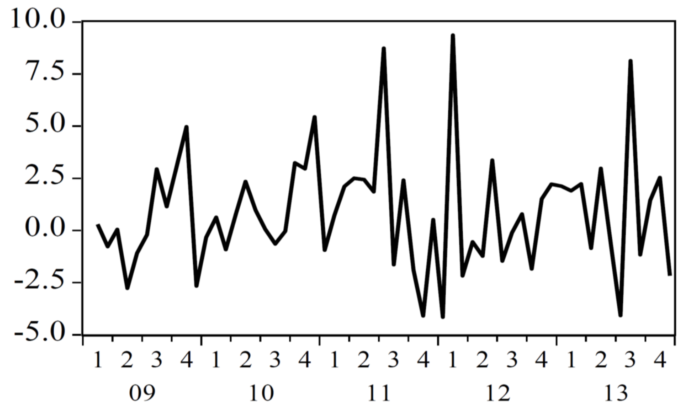
Consequently, based on the ADF and PP test results, it can be concluded that all series are nonstationary at level and stationary at first difference.

Furthermore, time plots for each seasonally adjusted and first differenced series are presented in Figure 3.3. In the figure, it can be clearly seen that there is no seasonality and upward or downward pattern with time, i.e., all the series are non-seasonal and stationary. Therefore, based on all the above methods of stationarity detection, the four time series are non-stationary originally and stationary after first differences. This implies all the series are integrated of order one ($I(1)$).

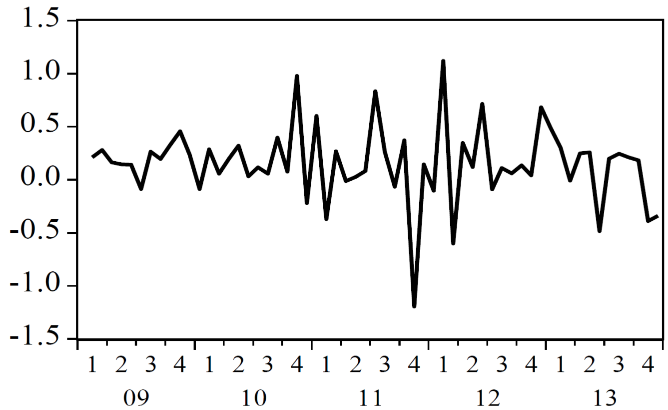
Plot of LF Adjusted versus Time (Ater first difference)
Y-Axis : First Differenced LF Adjusted and X-Axis : Time



Plot of PR Adjusted versus Time (Ater first difference)
Y-Axis : First Differenced PR Adjusted and X-Axis : Time



Plot of BH Adjusted versus Time (Ater first difference)
Y-Axis : First Differenced BH Adjusted and X-Axis : Time



Plot of DF Adjusted versus Time (Ater first difference)
Y-Axis : First Differenced DF Adjusted and X-Axis : Time

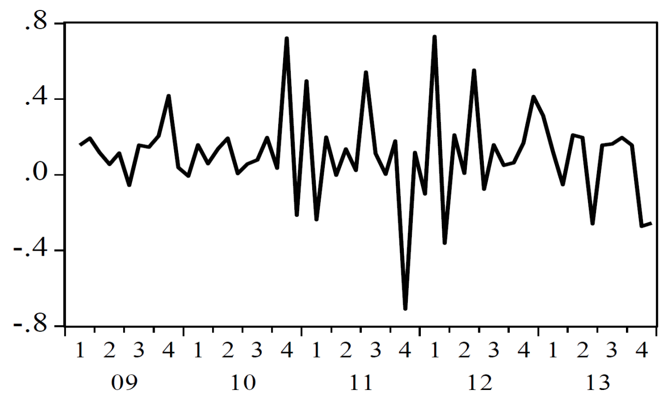


Figure 3.3. Time plots of seasonally adjusted series after first difference.

3.5. VAR model specification

3.5.1. Specification of VAR order

Determination of optimal lag order for the VAR/VEC model is done by using the Akaike information criterion (AIC), Schwartz information criterion (SC), and Hannan-Quin (HQ) information criterion. In each criterion, the lag with a minimum criterion value is selected as an optimum lag length for the model. The results are shown in Table 3.7 below.

Table 3.7. VAR lag order selection results

Lag	AIC	SC	HQ
0	1.477747	11.74201	11.88164
1	0.004127*	5.859841*	6.557956*
2	0.004457	5.929077	7.185684
3	0.005027	6.030597	7.845695
4	0.005274	6.042776	8.416366
5	0.005334	5.994265	8.926347
6	0.006327	6.072719	9.563293
7	0.007247	6.072470	10.12154
8	0.009315	6.128377	10.73593

*indicates lag order selected by the criterion.

From Table 3.7, the AIC, SC, and HQ test suggest the appropriate lag length for the VAR model to be one (1) since the minimum AIC, SC, and HQ values occur at lag one. Thus, it should be assumed that VAR(1) is the best for the data among all contender models.

3.5.2. Lag exclusion test

This test carries out confirmation for suitability of each lag selected by the above three criteria for the VAR. For each lag, the Chi square χ^2 (Wald) statistics of all variables are reported separately and jointly in Table 3.8 below.

Table 3.8. VAR lag exclusion Wald tests

Chi-squared test statistics for lag exclusion: Numbers in [ ] are p-values					
	LF	**PR**	**BH**	**DF**	**Joint**
Lag 1	10.67937	26.99636	25.87778	22.87876	80.94704
	[0.030414]	[1.99e-05]	[3.35e-05]	[0.000134]	[1.12e-10]
Lag 2	3.497042	4.116885	5.497757	5.484011	23.65859
	[0.478328]	[0.390418]	[0.239927]	[0.241139]	[0.097231]
Lag 3	4.213082	3.106088	4.182982	5.092473	26.19202
	[0.377936]	[0.540232]	[0.381808]	[0.277940]	[0.051386]
Lag 4	2.000285	8.778823	8.536231	9.098116	23.54798
	[0.735706]	[0.066872]	[0.073797]	[0.058693]	[0.099852]
Df	4	4	4	4	16

As it can be seen from Table 3.8 above, only the first lag is significant for LF at 5% significance level and for the remaining variables and for the joint at 1% significance level. Therefore, provided that VAR models usually need the same lag length for all the series, the Wald exclusion test assures that VAR(1) is found optimal for the data set and hence could be adopted.

3.5.3. Cointegration analysis

The idea behind cointegration analysis is that, although variables may tend to trend up and down over time, groups of variables may drift together. To determine the number of cointegrating relationships, the Johansen [13] approach of cointegration test is applied. The two tests for cointegration are the trace test and the maximum eigenvalue statistics. Here, these two test statistics are compared to special critical values to determine the number of cointegrating vector(s) in the model. The maximum eigenvalue and trace tests proceed sequentially from the first hypothesis – no cointegration – to an increasing number of co-integrating vectors.

From the results of Johansen cointegration test presented in Table 3.9 below, it can be observed that the trace or estimated LR statistic (63.35851) exceeds the respective critical value (47.85613) with P-value (0.0009). The maximum eigenvalue test also supports the same thing as the trace test. This implies that the null hypothesis of no cointegration relations is rejected at the 5% significance level in favour of the alternative one which states that there exists one cointegration relation. Therefore, the rank of cointegration matrix is equal to one, meaning, there is only one cointegrating equation in the system.

Table 3.9. Johansen cointegration test results (by assumption: Linear deterministic trend)

Hypothesised number of cointegration equation(s)	**Eigenvalue**	**Trace test**			**Maximum eigenvalue test**		
		Statistic	**Critical value (5%)**	**Prob.* ***	**Statistic**	**Critical value (5%)**	**Prob. ****
None *	0.222015	63.35851	47.85613	0.0009	38.66131	27.58434	0.0013
Atmost 1	0.099832	24.69720	29.79707	0.1726	16.19672	21.13162	0.2136
Atmost 2	0.051055	8.500472	15.49471	0.4135	8.070276	14.26460	0.3716
Atmost 3	0.002790	0.430196	3.841466	0.5119	0.430196	3.841466	0.5119

Normalized cointegrating coefficients (standard error in parentheses)

LF	PR	BH	DF
1.000000	– 0.540468	– 0.927617	0.944683
	(0.03997)	(0.09354)	(0.09852)
	[– 13.5210]	[– 9.91697]	[9.58913]

*denotes rejection of the hypothesis at the 0.05 level.

**MacKinnon-Haug-Michelis [20] p-values.

Consequently, the cointegrating vector is given by

$$\beta = (1, -0.540468, -0.927617, 0.944683).$$

The values correspond to the cointegrating coefficients of LF (normalized to one), PR, BH, and DF, respectively.

As far as the main purpose of cointegration analysis is to get a stationary series from two or more non-stationary series, the resulting stationary series is written as a linear combination of the nonstationary series under study. Accordingly, if this stationary series is designated by S_t, then using the results obtained from Table 3.9 above S_t is given by

$$S_t = LF_t - 0.540468PR_t - 0.927617BH_t + 0.944683DF_t, \quad (3.1)$$

(3.1) above enlightens that S_t is stationary in spite of the fact that all the four series are nonstationary.

3.6. Model estimation

After deduction is made then the variables in the VAR model appeared to be cointegrated, the immediate stride is to estimate the short-run behaviour and the adjustment to the long-run models, which is represented by VECM. The VEC model has the following structure:

$$\Delta Y_t = \mu + \sum_{i=1}^{p} \Gamma_i \Delta Y_{t-i} + \alpha BX_{t-1} + \varepsilon_t, \quad (3.2)$$

where BX_t is the error correction term given by $\beta' Y_t$ and β is the cointegrating vector. The responses of LF, PR, BH, and DF to short-term output movements are captured by the Γ_i coefficient matrices. The α coefficient vector reveals the speed of adjustments to the equilibrium, which measures the deviation from the long-run relationship between LF, PR, BH, and DF. Coefficient estimates of the VEC model are presented in Table 3.10 below. This long-run equilibrium model is:

$$LF_t = 47.27715 + 0.540468PR_t + 0.927617BH_t - 0.944683DF_t. \quad (3.3)$$

(3.3) above indicates that, in the long run, a one million dollar increase in the monthly Passenger Revenue accounts for an average increase of about 0.54% in the monthly Load Factor.

Table 3.10. Vector error correction estimates

Vector error correction estimates standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq.:	**Coint. Eq.1**
LF(-1)	1.000000
PR(-1)	– 0.540468
	(0.03997)
	[– 13.5210]
BH(-1)	– 0.927617
	(0.09354)
	[– 9.91697]
DF(-1)	0.944683
	(0.09852)
	[9.58913]
C	– 47.27715

Error correction:	**D(LF)**	**D(PR)**	**D(BH)**	**D(DF)**
Coint. Eq. 1	– 0.588086	0.662929	0.087086	0.607007
	(0.18673)	(0.20171)	(0.26189)	(0.18972)
	[– 3.14934]	[3.28654]	[0.33253]	[3.19943]
D(LF(-1))	– 0.652171	0.867706	0.384553	0.022715
	(0.21241)	(0.38976)	(0.45889)	(0.02388)
	[– 3.07039]	[2.22626]	[0.83801]	[0.95109]
D(PR(-1))	– 0.057621	– 0.278937	0.071514	0.005741
	(0.09491)	(0.16772)	(0.41041)	(0.01512)
	[– 0.60711]	[– 1.66314]	[0.17425]	[0.37975]
D(BH(-1))	0.757550	0.971061	– 0.920145	– 0.129807
	(0.187992)	(0.36552)	(0.56427)	(0.45777)
	[4.02969]	[2.65664]	[– 1.63068]	[– 0.28357]
D(DF(-1))	0.938681	0.952187	0.920495	– 0.114545
	(0.32081)	(0.34654)	(0.67738)	(0.67165)
	[2.92601]	[2.74773]	[1.35890]	[– 0.17054]
C	– 0.596773	0.062378	0.101714	0.123822
	(0.31018)	(0.26496)	(0.03383)	(0.03513)
	[– 1.92396]	[0.23543]	[3.00644]	[3.52441]

Table 3.10. (Continued)

R-squared	0.651586	0.680120	0.123152	0.168651
Adj. R-squared	0.619367	0.651933	0.102185	0.131674
Sum sq. resides	34.72280	423.5239	7.712675	3.441194
F-statistic	4.571581	4.109382	2.112504	2.190930
Log likelihood	– 68.72787	– 143.7643	– 23.59193	0.619467
Akaike AIC	3.853418	4.992143	0.986398	0.179351
Schwartz SC	4.062853	5.201577	1.195832	0.388786

Likewise, a one thousand hours flight time increase per month will result in an average increase by around 0.93% in the monthly Load Factor of the Ethiopian Aviation, in the long-run. In contrast, a one million kilometer increase in the monthly flight distance, on average in the long run, will come up with a decrease of about 0.94% in the load factor per month.

The second part of Table 3.10 contains the coefficients of the error correction terms (coint. Eq. 1) for the cointegration vector. These coefficients are called the *adjustment coefficients*. They measure the short-run adjustments of the deviations of the endogeneous variables from their long-run values. These first row coefficients identify the fraction of the long-term gap that is closed by each endogeneous variable in each period (months).

But before going to construct the individual VEC models, each variable should be checked whether they are endogeneous or exogeneous. This can be done through the following exogeneity test.

3.7. Granger causality/block exogeneity Wald test

This test detects whether the lags of one variable can Granger-cause any other variables in the VEC system. The null hypothesis is that all lags of one variable can be excluded from each VECM. Table 3.11 below presents an exogeneity test when each series are treated as dependent (endogeneous) variable.

Table 3.11. VEC granger causality/block exogeneity Wald tests

Depen dent variable : D(LF)			
Excluded	Chi-sq	df	Prob.
D(PR)	2.585663	1	0.1078
D(BH)	12.80246	1	0.0003
D(DF)	12.38657	1	0.0004
All	14.48618	3	0.0023
Dependent variable: D(PR)			
Excluded	Chi-sq	df	Prob.
D(LF)	6.208201	1	0.0127
D(BH)	8.623484	1	0.0033
D(DF)	8.236895	1	0.0041
All	9.579177	3	0.0020
Dependent variable: D(BH)			
Excluded	Chi-sq	df	Prob.
D(LF)	0.790258	1	0.3740
D(PR)	0.001542	1	0.9687
D(DF)	0.057172	1	0.8110
All	1.211434	3	0.7503
Dependent variable: D(DF)			
Excluded	Chi-sq	df	Prob.
D(LF)	1.043092	1	0.3071
D(PR)	4.76E-06	1	0.9983
D(BH)	0.007355	1	0.9317
All	1.797007	3	0.6156

Thus from the two parts of VEC estimates in Table 3.10, the following two VECMs can be straightforwardly estimated only for endogeneous variables (LF and PR) by introducing the error correction term as another independent variable in the restricted VAR model.

***VEC model of load factor*:**

$$\Delta LF = -\,0.59[LF_{t-1} - 0.54PR_{t-1} - 0.93BH_{t-1} + 0.94DF_{t-1} - 47.3]$$

$$-\,0.65\Delta LF_{t-1} - 0.06\Delta PR_{t-1} + 0.76\Delta BH_{t-1} + 0.94\Delta DF_{t-1} - 0.59; \quad (3.4)$$

***VEC model of passenger revenue*:**

$$\Delta PR = 0.66[LF_{t-1} - 0.54PR_{t-1} - 0.93BH_{t-1} + 0.94DF_{t-1} - 47.3] + 0.87\Delta LF_{t-1}$$

$$-\,0.28\Delta PR_{t-1} + 0.97\Delta BH_{t-1} - 0.95\Delta DF_{t-1} + 0.062, \quad (3.5)$$

where "□" denotes first difference (D), the value in the closed bracket is the error correction term and the coefficients of error correction term are called *adjustment coefficients*.

Therefore, from (3.4) and (3.5) above it can be realized that, each month, 59% and 66% of the long term gaps are closed by LF and PR, respectively. That is, 59% and 66% of the short run disequilibria in LF and PR are adjusted within one month, respectively. On the other hand, the long term BH and DF gaps are closed by about 8.7% and 61% in each month as it can be referred from Table 4.10, respectively. It is also possible to say that 8.7% and 61% of the short run disequilibria in BH and DF are adjusted within a single month. These results imply that BH and PR have the shortest and longest speed, respectively, to get back to the equilibrium after a shock. LF and DF share almost equal speed to achieve equilibrium after a shock. Additionally, LF is significantly affected by BH, DF and its own lagged values in the short-run. On its part, PR is significantly determined by lagged values of all the variables, except its own lagged values, in the short run. However, BH and DF are insignificantly affected by all of the variables in the short run.

3.8. Model checking

Subsequent to model development, it is necessary to verify whether the fitted model is suitable. All the time it is after model validity examination that forecasting will be made.

3.8.1. Test of residual autocorrelation

Table 3.12 below presents the results of the Portmanteau Q-statistic and Lagrange Multiplier (LM) test for the whole VEC model residual serial correlation.

Table 3.12. Test of residual autocorrelation

Lag	Q-Stat		Adj Q-Stat		LM-Stat	
	Value	Prob.	Value	Prob.	Value	Prob.
1	2.432637	NA*	2.473868	NA*	14.38020	0.5704
2	18.54117	0.9117	19.13787	0.8938	19.07306	0.2649
3	42.37984	0.5412	44.23121	0.4619	22.35275	0.1322
4	54.94693	0.6604	57.69594	0.5604	13.66433	0.6237
5	70.40964	0.6593	74.56435	0.5251	16.42390	0.4238
6	87.10019	0.6249	93.10941	0.4481	17.37960	0.3615
7	101.1300	0.6672	108.9922	0.4552	15.29466	0.5032
8	108.5863	0.8364	117.5957	0.6448	8.167734	0.9437
9	121.3304	0.8706	132.5887	0.6594	15.63678	0.4786
10	132.5038	0.9139	145.9968	0.7057	13.76859	0.6159
11	147.6776	0.9103	164.5769	0.6443	23.66651	0.0970
12	164.7226	0.8886	185.8831	0.5300	21.16612	0.1722

***The test is valid only for lags larger than the VAR lag order.**

The tests in Table 3.12 above are used to test for the overall significance of the residual autocorrelations up to lag 12. Both tests imply that residuals do not suffer from autocorrelation problem up to lag 12 as all p-values go beyond the 5% level of risk.

3.8.2. Testing normality

Multivariate version of the Jarque-Bera tests is used to test the normality of the residuals. It compares the 3rd and 4th moments (skewness and kurtosis) to those from a normal distribution. The test has null hypothesis indicating that the error term in the model has skewness and kurtosis corresponding to a normal distribution.

Table 3.13. Normality test

Component	Skewness		Kurtosis		Jarque-Bera Statistic	
	Value	Prob.	Value	Prob.	Value	Prob.
1	– 0.512526	0.1051	4.331375	0.0353	7.058224	0.0293
2	0.409035	0.1958	3.384622	0.5431	2.042931	0.3601
3	– 0.616863	0.0511	3.445014	0.4817	4.300290	0.1165
4	– 0.083268	0.7923	2.474528	0.4061	0.759639	0.6840
Joint		0.0854		0.2001		0.077

The results in Table 3.13 show that there is no evidence to reject the null hypothesis of normality for the whole VEC model residuals.

3.9. Structural analysis

3.9.1. Granger causality test

Table 3.14 below presents results from the pair wise Granger causality tests at 5% significance level. The result shows that at 95% confidence level, Block hours (BH) and Distance flown (DF) Granger cause the Load factor (LF) but the converses do not hold. Passenger revenue (PR) does not Granger cause LF. That is, only the change in PR does not account for the change in LF. Beside, PR is Granger caused by all the variables but the reverses fail. That is, the changes in all variables will result in the change in PR.

Table 3.14. Pairwise Granger causality tests

Null Hypothesis:	Obs.	F-Statistic	Prob.
PR does not Granger Cause LF	59	0.22910	0.6341
LF does not Granger Cause PR		7.32725	0.0090
BH does not Granger Cause LF	59	11.6683	0.0008
LF does not Granger Cause BH		2.98006	0.0863
DF does not Granger Cause LF	59	11.4995	0.0009
LF does not Granger Cause DF		2.50532	0.1155
BH does not Granger Cause PR	59	13.5908	0.0003
PR does not Granger Cause BH		0.00248	0.9604
DF does not Granger Cause PR	59	13.8178	0.0003
PR does not Granger Cause DF		0.00616	0.9375
DF does not Granger Cause BH	59	1.74584	0.1884
BH does not Granger Cause DF		0.05641	0.8126

3.9.2. Impulse-response functions

Impulse responses trace out the reaction of the variables in the VAR to shocks of each variable. Therefore, for each variable a unit shock is applied to the error and the effects upon the VAR system over time are noted.

The x-axis in Figure A1((a) and (b)) in the Appendix part provides the time horizon or the duration of the shock whilst the y-axis gives the direction and intensity of the impulse or the percent variation in the dependent variable away from its base line level. In our case there are 8 potential impulse response functions. The outcomes and combined graphs of these IRF functions are given in Table A5 ((a) and (b)) and Figure A1 ((a) and (b)) of Appendix with the Cholesky ordering of LF, PR, BH, and DF, respectively.

Figure A1(a) shows the responses of LF, PR, BH, and DF with respect to one standard deviation innovation in LF. The result indicates LF innovations have a positive impact on PR and BH. For PR, it displays a slow rising trend until it reaches 0.072 intensity value and it calms down at around 2 month time horizon. Similarly, the shocks of LF show a positive effect upon BH with a fast intensifying up to an impulse intensity of 0.23, then declines rapidly to an intensity of 0.092 and finally stabilizes at 3 month time horizon. But the one standard shocks in LF have totally a negative impact on DF. This impact initially exhibits a slight decrease up to a – 0.2 and becomes constant after 4 month time horizon.

Figure A1(b) shows that the effects of a one standard deviation shock in PR on the remaining variables. From the figure, the shocks have a positive response for LF. That is, it reveals a sluggish diminishing pattern up to 1.11 level of intensity and moves upward moderately until it becomes steady at around 3. The shocks have also a parallel effect in BH but an opposite response in DF.

3.9.3. Forecast error variance decomposition

The decomposition results of the models of endogeneous variables (LF and PR) are plotted in Figure A2((a) and (b)) of Appendix. These two results provide the forecast error percentage in each variable that could be attributed to innovations of the other variables, for different time periods. The Cholesky ordering employed is LF, PR, BH, and DF.

The variance decomposition analysis result of Figure A2(a) shows that, at the first horizon, variation of LF is explained only by its own shock. In the second month 97.83% of the variability in the LF fluctuations is explained by its own innovations and the remaining 2.17% is explained by BH (1.71%), DF (0.29%), and PR (0.17%). Even up to the tenth month, much of the variability of LF (85.93%) is explained by its own shock and the rest portion is occupied by DF (8.74%), BH (4.01%), and PR (1.32%). It can also be observed that, after ten months, the variability of LF determined by DF has shown an increment to 8.74% and the LF shock revealed a total of 14.07% decrement. However, the percentages of BH and PR to LF variability explanation seem to never increase beyond 5% and 2%, respectively, even after large amount of duration.

In a similar fashion, Figure A2(b) displays that, in the first month, 81.25% of the variability of PR is explained by its own shock and 18.75% is determined by LF. After ten months the variability of PR explained by its shocks and LF attained 70.05% and 26.85%, respectively.

3.10. Forecasting

This section conducts an examination on the forecasting accuracy of the fitted model and then makes a forecast for January 2014 to December 2014. Meaning, one year ahead forecast is made and can be seen from Table 3.16.

3.10.1. Evaluation of accuracy

The mean square error (MSE), root mean square error (RMSE), mean absolute error (MAE), and Theil U statistics are used to assess the forecasting performance. In evaluating the performance of the forecasting models, the lower the RMSE, MAE, MAPE, and Theil-U statistic, the better the forecasting accuracy.

Table 3.15. Forecasting accuracy statistics

Forecast sample: January 2013 to December 2013		
Accuracy measures	**Variables**	
	LF	**PR**
Root mean squared error	1.090529	4.369759
Mean absolute error	0.891087	3.970237
Mean absolute percent error	1.249139	3.931896
Theil inequality coefficient	0.007653	0.022276

3.10.2. Out-of-sample and in-sample forecasting analysis

Out-of-sample forecasted values for the series under study, using the vector error correction model, are presented in Table 3.16 below.

Table 3.16. Forecasts from the VECM (1) models

Months		LF	PR
January	**2014**	71.39764	92.39132
February	**2014**	71.31635	93.70771
March	**2014**	71.47449	94.64862
April	**2014**	71.53258	93.66481
May	**2014**	71.78363	94.04010
June	**2014**	71.53519	95.94809
July	**2014**	71.22674	98.67355
August	**2014**	71.18153	99.72597
September	**2014**	70.76529	102.4194
October	**2014**	70.95727	103.9212
November	**2014**	70.94412	104.6392
December	**2014**	70.77500	105.7876

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■

Appendix

Table A(1). Seasonality F-tests and adjustment quality diagnostics of original series

Table A1(a). F-tests for seasonality and adjustment quality diagnostics of original PR

Test for the presence of seasonality assuming stability:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	7302.5129	11	663.86481	130.619**
Residuals	243.9578	48	5.08246	
Total	7546.4707	59		

****Seasonality present at the 0.1 per cent level**

Nonparametric test for the presence of seasonality assuming stability:

	Kruskal-Wallis statistic	Degrees of freedom	Probability level
	56.3639	11	0.000%

Seasonality present at the one percent level

Moving seasonality test:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	24.8880	4	6.221996	1.542
Error	177.5488	44	4.035201	

No evidence of moving seasonality at the five percent level

COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY: IDENTIFIABLE SEASONALITY PRESENT

Test for the presence of residual seasonality:

No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.09

No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.56

No evidence of residual seasonality in the last 3 years at the 5 percent level.

M1 = 0.099, M2 = 0.113, M3 = 0.000, M4 = 0.201, M5 = 0.225, M6 = 0.521, M7 = 0.211, Q = 0.19

Table A1(b). F-tests for seasonality and adjustment quality diagnostics of original BH

Test for the presence of seasonality assuming stability:				
	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	1594.6490	11	144.96809	54.912**
Residuals	126.7196	48	2.63999	
Total	1721.3686	59		
****Seasonality present at the 0.1 percent level**				
Nonparametric test for the presence of seasonality assuming stability:				
	Kruskal-Wallis statistic	Degrees of freedom	Probability level	
	53.4551	11	0.000%	
Seasonality present at the one percent level				
Moving seasonality test:				
	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	9.5947	4	2.398663	0.948
Error	111.3712	44	2.531163	
No evidence of moving seasonality at the five percent level				
COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY: IDENTIFIABLE SEASONALITY PRESENT				
Test for the presence of residual seasonality:				
No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.23				
No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.80				
No evidence of residual seasonality in the last 3 years at the 5 percent level.				
M1 = 0.182, M2 = 0.194, M3 = 0.000, M4 = 0.161, M5 = 0.147, M6 = 0.056, M7 = 0.299, Q = 0.18				

Table A1(c). F-tests for seasonality and adjustment quality diagnostics of original DF

Test for the presence of seasonality assuming stability:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	1690.9310	11	153.72100	59.340**
Residuals	124.3450	48	2.59052	
Total	1815.2760	59		

****Seasonality present at the 0.1 percent level**

Nonparametric test for the presence of seasonality assuming stability:

Kruskal-Wallis statistic	Degrees of freedom	Probability level
53.2098	11	0.000%

Seasonality present at the one percent level

Moving seasonality test:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	7.9251	4	1.981283	0.863
Error	101.0621	44	2.296867	

No evidence of moving seasonality at the five percent level

COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY: IDENTIFIABLE SEASONALITY PRESENT

Test for the presence of residual seasonality:

No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.15

No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.67

No evidence of residual seasonality in the last 3 years at the 5 percent level.

M1 = 0.200, M2 = 0.214, M3 = 0.000, M4 = 0.282, M5 = 0.144, M6 = 0.114, M7 = 0.284, Q = 0.20

Table A2. Post-seasonal adjustment tests

Table A2(a). F-tests for seasonality of passenger revenue series after adjustment

Test for the presence of seasonality assuming stability:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	24.0917	11	2.19016	0.514
Residuals	204.7018	48	4.26462	
Total	228.7936	59		

No evidence of stable seasonality at the 0.1 percent level

Nonparametric test for the presence of seasonality assuming stability:

	Kruskal-Wallis statistic	Degrees of freedom	Probability level
	3.7502	11	97.666%

No evidence of seasonality at the one percent level

Moving seasonality test:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	1.2852	4	0.321289	0.135
Error	**104.3351**	44	2.371253	

No evidence of moving seasonality at the five percent level

COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY: IDENTIFIABLE SEASONALITY PRESENT

Test for the presence of residual seasonality:

No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.10

No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.49

No evidence of residual seasonality in the last 3 years at the 5 percent level.

M7 = 2.189

Table A2(b). F-tests for seasonality of block hours series after adjustment

Test for the presence of seasonality assuming stability:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	7.9035	11	0.71850	0.316
Residuals	109.2417	48	2.27587	
Total	117.1452	59		

No evidence of stable seasonality at the 0.1 percent level

Nonparametric test for the presence of seasonality assuming stability:

Kruskal-Wallis statistic	Degrees of freedom	Probability level
1.4590	11	99.967%

No evidence of seasonality at the one percent level

Moving seasonality test:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	3.3667	4	0.841686	0.825
Error	**44.8811**	44	1.020025	

No evidence of moving seasonality at the five percent level

COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY: IDENTIFIABLE SEASONALITY NOT PRESENT

Test for the presence of residual seasonality:

No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.19

No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.65

No evidence of residual seasonality in the last 3 years at the 5 percent level.

M7 = 3.000

Table A2(c). F-tests for seasonality of distance flown series after adjustment

Test for the presence of seasonality assuming stability:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between months	8.0585	11	0.73259	0.311
Residuals	113.0569	48	2.35535	
Total	121.1154	59		

No evidence of stable seasonality at the 0.1 percent level

Nonparametric test for the presence of seasonality assuming stability:

Kruskal-Wallis statistic	Degrees of freedom	Probability level
1.1508	11	99.990%

No evidence of seasonality at the one percent level

Moving seasonality test:

	Sum of squares	Degrees of freedom	Mean square	F-value
Between years	1.0636	4	0.265898	0.244
Error	47.9685	44	1.090194	

No evidence of moving seasonality at the five percent level

COMBINED TEST FOR THE PRESENCE OF IDENTIFIABLE SEASONALITY: IDENTIFIABLE SEASONALITY NOT PRESENT

Test for the presence of residual seasonality:

No evidence of residual seasonality in the entire series at the 1 percent level. F = 0.13

No evidence of residual seasonality in the last 3 years at the 1 percent level. F = 0.59

No evidence of residual seasonality in the last 3 years at the 5 percent level.

M7 = 2.980

Table A5(a). Impulse response results (Cholesky ordering: LF, PR, BH, DF)

Period	LF	PR	BH	DF
1	1.584788	0.000000	0.000000	0.000000
2	0.707372	0.072319	0.229310	– 0.095068
3	0.570479	0.060804	0.091592	– 0.144731
4	0.351148	0.082558	0.141682	– 0.203685
5	0.306381	0.079909	0.124629	– 0.213003
6	0.253537	0.084614	0.128889	– 0.226599
7	0.235466	0.084852	0.127147	– 0.231316
8	0.222716	0.085738	0.127576	– 0.234435
9	0.216955	0.085959	0.127282	– 0.235927
10	0.213540	0.086162	0.127334	– 0.236781

Table A5(b). Response of PR

Period	LF	PR	BH	DF
1	1.212795	2.524314	0.000000	0.000000
2	1.113426	2.150071	0.508846	– 0.420855
3	1.445829	2.163550	0.443438	– 0.145376
4	1.378483	2.171589	0.399866	– 0.204049
5	1.393663	2.169909	0.429614	– 0.199768
6	1.412428	2.167921	0.419615	– 0.191979
7	1.410077	2.168827	0.421929	– 0.193672
8	1.414102	2.168293	0.421591	– 0.192521
9	1.414713	2.168361	0.421675	– 0.192317
10	1.415383	2.168301	0.421639	– 0.192181

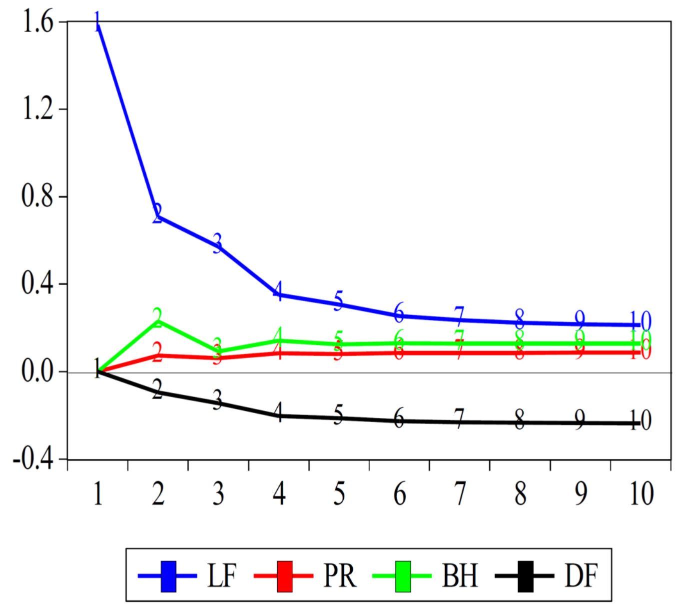

Figure A1(a). Response of LF to Cholesky One S.D. innovations.

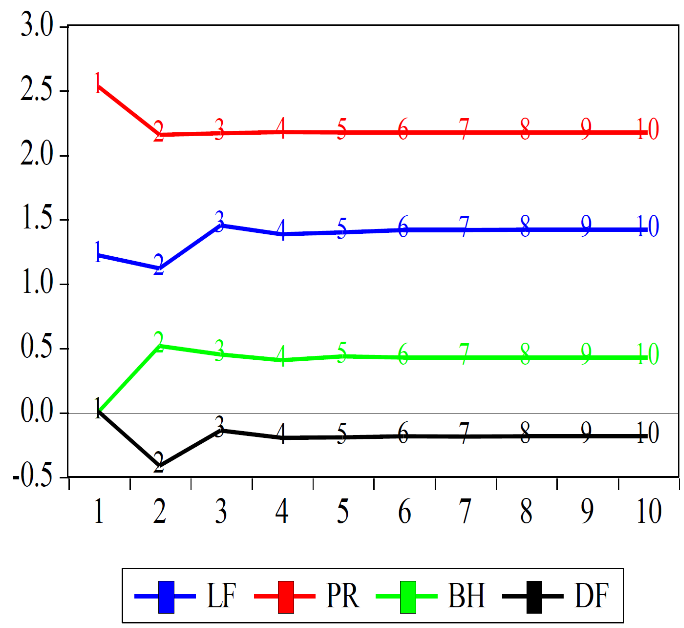

Figure A1(b). Response of PR to Cholesky One S.D. innovations.

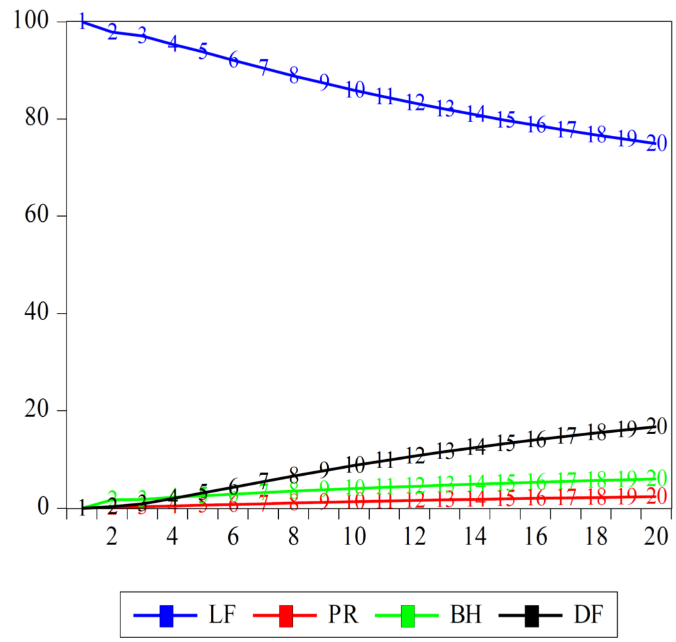

Figure A2(a). Variance decomposition of LF.

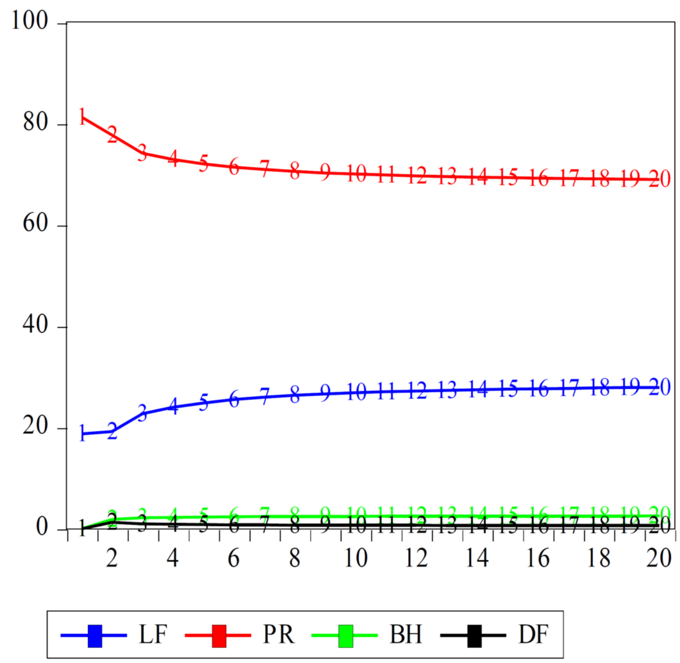

Figure A2(b). Variance decomposition of PR.