International Journal of Statistics: Advances in Theory and Applications
Vol. 1, Issue 1, 2020, Pages 111-120
Published Online on August 16, 2019

http://jyotiacademicpress.org

ARCSINE CUMULATIVE SUM CONTROL CHART

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Abstract

Recent applications of control charts show that they are capable of monitoring not only manufacturing processes but also service processes. Data sets generated by several service processes often come from a process with non-normal distribution or unknown distribution. As most of the existing charts are not suitable to monitor such non-normal processes, in this paper, a new type of 'Cumulative Sum (CUSUM)' chart based on arcsine transformation over a simple statistic is proposed to monitor the shifts of the process mean of a few service processes. Using the sampling properties of the new monitoring statistic, the average run lengths of the proposed chart have been calculated to check if the chart (i) is capable of detecting both small and large shifts in the process mean and (ii) performs as good as the transformed EWMA (exponentially weighted moving average) chart due to Su-Fen et al. ([9]) to monitor processes that exhibit a drifting mean over time. A numerical example of service times with skewed distribution from a service system of a bank in Taiwan is used to illustrate the performance of this chart.

2020 Mathematics Subject Classification: 62P30, 00A06.
Keywords and phrases: in-control, arcsine, cusum, statistical process control, nonconforming.
Received May 1, 2017

1. Introduction

The main objective of statistical process control (SPC) is to quickly detect the presence of assignable causes of variation so that corrective action can be taken to remove them before many nonconforming units are manufactured. One of the tools used in SPC is a control chart, which has proved to be very good in improving productivity, providing diagnostics information about the process and, useful for detecting and preventing unnecessary process adjustment. Control charts are also becoming effective tools in improving service quality as demonstrated by Su-Fen et al. [9]; MacCarthy and Wasusri [7]; and Tsung et al. [10].

The application of control charts mainly assumes that the process output follows a normal distribution. However, most of the service process data violates this assumption as most of them come from either non-normal or unknown distributions. According to Su-Fen et al., when we have no knowledge of the underlying distribution, it is not possible to derive the necessary sampling properties to construct the Shewhart charts and evaluate their performance. To overcome this difficulty, researchers like Amin et al. [1]; Chakraborti et al. [4]; Chakraborti and Eryilmaz [5] as well as Chakraborti and Graham [6] proposed several nonparametric control charts for process data.

However, Su-Fen et al. [9] observed that these nonparametric control charts are not easy for practitioners to apply because the practitioners are not statisticians and do not understand the proper way of implementing the charts. Thus, the authors proposed a simple to understand and easy to use Shewhart type control chart, mean chart and an arcsine transformed EWMA control chart for variable data to monitor process mean of service process data without assuming a process distribution. The proposed EWMA chart is more effective in detecting both small and large shift in the process mean than the mean chart.

2. The New CUSUM Chart

In this article, we propose a new arcsine transformed cumulative sum (CUSUM) control chart for monitoring process mean for service process data without assuming a process distribution.

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample of n observations from the process with a process mean μ.

Define $Y_j = X_j - \mu$ and $p = P(Y > 0)$ as the process proportion.

If the process is in-control, then $p = p_o$. And, $p \neq p_o$ when the process is out-of-control. Let $I_j = \begin{cases} 1 \text{ if } Y_j > 0, \\ 0 \text{ otherwise.} \end{cases}$

Further let M_i be the total number of $Y_j > 0$ in the i-th sample. Then, $M_i = \sum_{j=1}^{n} I_j$ would follow a binomial distribution with parameter (n, p_0) when the process is in-control Su-Fen et al. [9]. "M_i" is the number of X_i values in the sample that are above the target process mean.

Let $T_i = \sin^{-}\left(\sqrt{\frac{M_i}{n}}\right)$. Then the distribution of T_i would be approximately normal with mean $\sin^{-}(\sqrt{p})$, where $p = P(Y > 0)$ and variance $1/4n$ (Mosteller and Youtz [8]). Thus, the CUSUM statistics based on T_i is given as

$$C_i^+ = \max[0, T_i - (\sin^{-}(\sqrt{p}) + k) + C_{i-1}^+],$$

$$C_i^- = \max[0, (\sin^{-}(\sqrt{p}) - k) - T_i + C_{i-1}^-],$$

where k, the reference value, is half the shift the chart is intended to detect. With a decision interval $h = 5\sigma$, we plot the C_i^+ and C_i^- in a control chart with the control limit given as h and, if any of the two statistics plots above h, the process is deemed to be out-of-control.

The average run length (ARL) of a control chart is often used as the sole measure of performance of the chart. The ARL of the chart is the average number of points that must be plotted before a point plots above the decision interval. If this happens, an out-of-control signal is issued and a search for an assignable cause(s) of variation must be mounted. A chart is considered to be more efficient if its ARL is smaller than those of all other competing charts when the process is out-of-control and the largest when the process is in-control.

The out-of-control signal is issued when either C_i^+ or C_i^- plots above the decision interval. Therefore, the plan (the sample size and control limits) is chosen so that the ARL is large when the process is in-control and small when the process is out-of-control. Cox [3] suggested that the criteria for a good chart are acceptable risks of incorrect actions, expected average quality levels reaching the customer and expected average inspection loads. Therefore, the in-control ARL should be chosen so as to minimize the frequency of false alarms and to ensure adequate response times to genuine shifts.

For a predetermined in-control ARL, for quickly detecting shifts in the mean, an optimal combination of h and k is determined which will minimize the out-of-control ARL for a specified change (Cheng and Thaga [2]). For these values, (ARL, k) the value of the decision interval (h) follows for various changes in the process mean. Since there is no direct way to compute the ARL, each ARL value is obtained by using 10,000 simulations. The ARLs for this chart are shown in Table 1 and its performance in detecting shifts in the process mean is compared with the mean and EWMA charts proposed by Su-Fen et al. [9] with an in-control average run length ARL_0 set at 370.5 runs. The results show that the arcsine CUSUM and arcsine EWMA charts are more sensitive to both small and large shifts in the process mean in comparison to the mean chart with the EWMA chart being slightly performing better than the arcsine CUSUM. For example, an arcsine EWMA chart will detect a shift in the process proportion from 0.613 to 0.85 on the 4-th sample while the arcsine CUSUM detects it on the 18-th sample if we take samples of size 9 while the mean chart will only detect that shift after taking more than 2 million samples. The two charts are more sensitive than the mean chart, particularly for small sample where the mean chart is not sensitive to any shift. For large samples (more than 20), the mean chart performs better than these two charts for large process mean shifts.

Table 1. Performance of the arcsine CUSUM chart against mean chart and arcsine EWMA charts

$ARL_0 = 370.5$ and $p_0 = 0.613$						
Chart	n	P				
		0.55	0.65	0.75	0.85	0.95
Mean Chart	9	1321.6	12687.7	262144	26012295	5.12E+11
Arcsine CUSUM		185.5	210.4	53.9	17.5	7.4
EWMA		132.0	153.8	15.1	3.4	1.5
Mean Chart	10	2936.8	36251.0	1048576	173415306	1.024E+13
Arcsine CUSUM		166.1	204.3	49.6	15.8	6.8
EWMA		123.0	146.8	13.3	3.1	1.4
Mean Chart	14	464.9	7087.2	311410	1.141E+8	4947E+13
Arcsine CUSUM		104.7	184.0	37.2	11.5	5.7
EWMA		96.2	123.6	8.7	2.3	1.3
Mean Chart	15	810.0	617.8	74.8	11.4	2.2
Arcsine CUSUM		94.3	179.8	34.9	10.8	5.6
EWMA		91.1	118.8	8.0	2.1	1.2
Mean Chart	20	609.0	458.1	41.1	5.7	1.4
Arcsine CUSUM		85.4	162.1	26.8	8.4	5.2
EWMA		71.8	96.1	5.4	1.7	1.1

3. Example

An arcsine CUSUM chart is applied to real data obtained from Su-Fen et al. [9]. The service time is an important quality characteristic for a bank branch in Taiwan. To measure the efficiency in the service system of a bank branch, the sampling service times (in minutes) are measured from 10 counters every 2 days for 30 days, that is, 15 samples of 10. These data have been analysed and have a right-skewed distribution. Data are given in Table 2.

Table 2. Service time from 10 counters in a bank branch

Sample No.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
1	0.88	0.78	5.06	5.45	2.93	6.11	11.59	1.20	0.89	3.21
2	3.82	13.4	5.16	3.20	32.27	3.68	3.14	1.58	2.72	7.71
3	1.40	3.89	10.88	30.85	0.54	8.40	5.10	2.63	9.17	3.94
4	16.8	8.77	8.36	3.55	7.76	1.81	1.11	5.91	8.26	7.19
5	0.24	9.57	0.66	1.15	2.34	0.57	8.94	5.54	11.69	6.58
6	4.21	8.73	11.44	2.89	19.49	1.20	8.01	6.19	7.48	0.07
7	15.08	7.43	4.41	6.14	10.37	2.33	1.97	1.08	4.27	14.08
8	13.89	0.30	3.21	11.32	9.90	4.39	10.5	1.70	10.74	1.46
9	0.03	12.76	2.41	7.41	1.67	3.70	4.31	2.45	3.57	3.33
10	12.89	17.96	2.78	3.21	1.12	12.61	4.23	6.18	2.33	6.92
11	7.71	1.05	1.11	0.22	3.53	0.81	0.41	3.73	0.08	2.55
12	5.81	6.29	3.46	2.66	4.02	10.95	1.59	5.58	0.55	4.10
13	2.89	1.61	1.30	2.58	18.65	10.77	18.23	3.13	3.38	6.34
14	1.36	1.92	0.12	11.08	8.85	3.99	4.32	1.71	1.77	1.94
15	21.52	0.63	8.54	3.37	6.94	3.44	3.37	6.37	1.28	12.83

Just like the mean and arcsine EWMA charts, the arcsine CUSUM chart shows that the process is in control as shown in Figure 1 with all data points plotting below the decision interval.

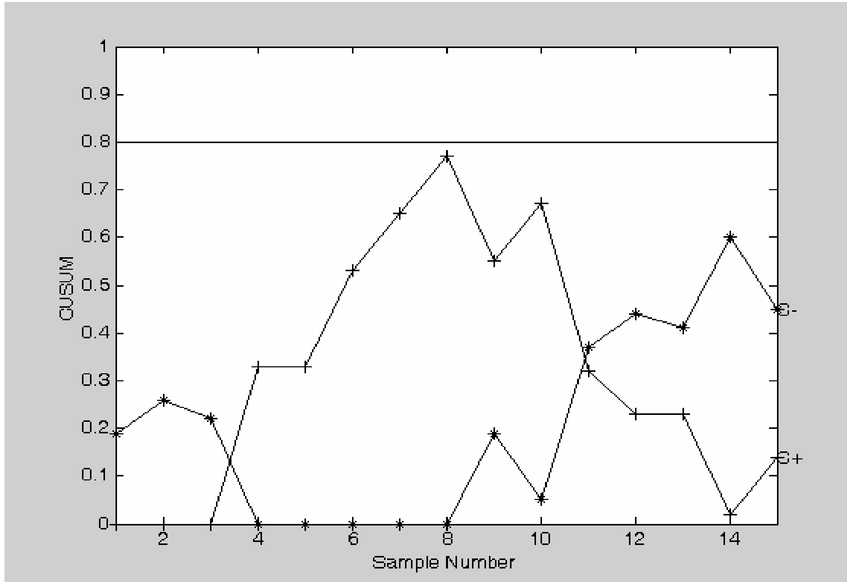

Figure 1. Arcsine CUSUM chart for service time.

To show the detection ability of the proposed arcsine CUSUM chart, 10 additional samples from a new automatic service system from the bank branches were collected and the data is shown in Table 3. The control chart used to monitor the old process was used to monitor the new process. The results in Figure 2 show that the process immediately got out-of control on the 16-th sample. The new improved system significantly reduces the service time. The EWMA chart proposed by Su-Fen et al. [9] had detected the shift also in the 16-th sample while the mean chart detected it on the 17-th sample.

Table 3. Service times from new automatic system from the 10 counters in a bank branch

Sample No.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
16	3.54	0.01	1.33	7.27	5.52	0.09	1.84	1.04	2.91	0.63
17	0.86	1.61	1.15	0.96	0.54	3.05	4.11	0.63	2.37	0.05
18	1.45	0.19	4.18	0.18	0.02	0.70	0.80	0.97	3.60	2.94
19	1.37	0.14	1.54	1.58	0.45	6.01	4.59	1.74	3.92	4.82
20	3.00	2.46	0.06	1.80	3.25	2.13	2.22	1.37	2.13	0.25
21	1.59	3.88	0.39	0.54	1.58	1.70	0.68	1.25	6.83	0.31
22	5.01	1.85	3.10	1.00	0.09	1.16	2.69	2.79	1.84	2.62
23	4.96	0.55	1.43	4.12	4.06	1.42	1.43	0.86	0.67	0.13
24	1.08	0.65	0.91	0.88	2.02	2.88	1.76	2.87	1.97	0.62
25	4.56	0.44	5.61	2.79	1.73	2.46	0.53	1.73	7.02	2.13

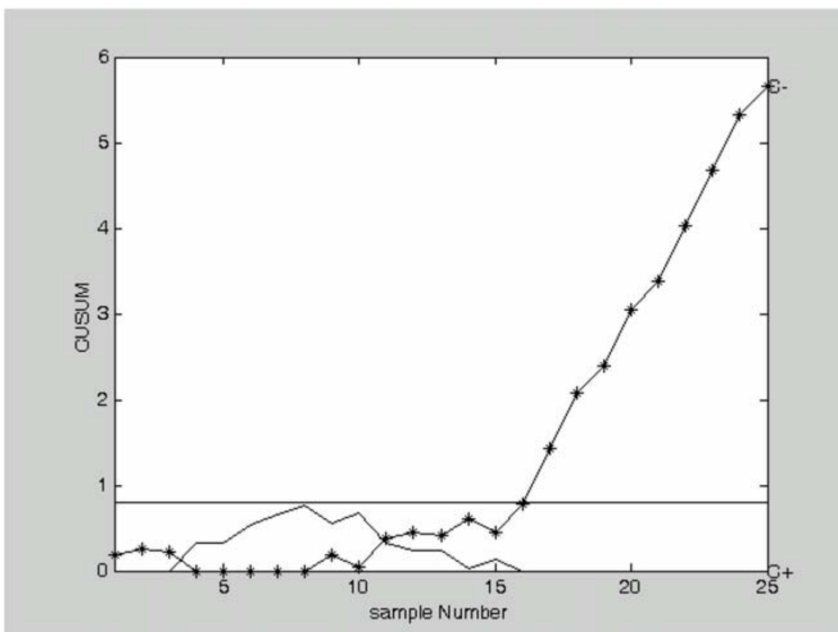

Figure 2. Arcsine CUSUM chart for new automatic system.

4. Conclusion

In this article, a new type of the mean chart is proposed to monitor the mean shifts in the process. As the new monitoring statistic is obtained by arcsine transformation ensures normal approximations, the ARLs of the proposed chart are calculated by using properties of the normal distribution. The detection ability of the proposed CUSUM chart was compared with two more existing charts. The new CUSUM chart showed better detection ability than the mean chart and is capable of detecting both small and large shifts in the process mean particularly for large samples.

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