

**COMPARING THE DESCRIPTIVE  
CHARACTERISTICS OF THE ADJUSTED SUCCESS  
RATE TO THE PHI COEFFICIENT, THE ODDS  
RATIO, AND THE DIFFERENCE BETWEEN  
CONDITIONAL PROPORTIONS**

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**Abstract**

At present, there are three simple ways to describe the association of between participants binary variables. One of them is to use the phi coefficient. Another is to use the odds ratio or similar method (e.g., risk ratio or absolute risk reduction), possibly supplemented by Yule's  $Q$ . A third method is to use the difference between conditional proportions. I propose another simple alternative, the adjusted success rate, not to replace these methods but as an addition to the researcher's toolbox. I also compare and contrast the three methods with the adjusted success rate.

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### 1. Introduction

Researchers are sometimes in the position of desiring to assess the association between between-participants binary variables in the form of a  $2 \times 2$  table. Table 1 is an illustration of the association between binary variables where  $a$ ,  $b$ ,  $c$ , and  $d$  represent the cell frequencies, row 1 and row 2 represent the row margin frequencies, and column 1 and column 2 represent the column margin frequencies. The number of cases in total ( $N$ ) equals the sum of the cell frequencies ( $N = a + b + c + d$ ).

The phi coefficient ( $r_\phi$ ) is one way to describe the association between two binary variables as is illustrated by Table 1. The phi coefficient is rendered as Equation (1) below (see Siegel & Castellan [6] for a review).

$$r_\phi = \frac{|ad - bc|}{\sqrt{(a + b)(c + d)(a + c)(b + d)}}. \quad (1)$$

Under the condition that the margin frequencies are equivalent, the phi coefficient is very useful and easily interpretable because it ranges from 0 (no association) to 1 (one variable is completely predictable from the other). However, an important disadvantage is that matters change when the margin frequencies are not equivalent. For example, suppose that the margin frequencies are 100 and 100 for *row 1* and *row 2*, respectively, but that the margin frequencies are 180 and 20 for *column 1* and *column 2*, respectively. In this case, the maximum phi coefficient that could be obtained is .33 rather than unity. Suppose that the cell frequencies in this case are as follows:  $a = 99$ ,  $b = 1$ ,  $c = 81$ , and  $d = 19$  so that the phi coefficient is .3. The interpretation is not completely clear.

**Table 1.** The production of cell frequencies in a  $2 \times 2$  table with binary variables

Variable 1	Variable 2		Row Margins
	0	1	
0	$a$	$b$	$a + b$ , row 1
1	$c$	$d$	$c + d$ , row 2
Column Margins	$a + c$	$b + d$	
	column 1	column 2	

The odds ratio ( $OR$ ) provides another way to describe the association between binary variables (see Lipsey & Wilson [4] for a review). The researcher can describe a particular outcome as a “success” and so the odds ratio is the odds of a success in one group divided by the odds of a success in the other, in accordance with Equation (2) below where  $p_1$  gives the probability of a success in one group and  $p_2$  gives the probability of a success in the other group. Alternatively, the odds ratio can be given in terms of the four cells, as also is expressed in Equation (2).

$$OR = \frac{\frac{p_1}{1-p_1}}{\frac{p_2}{1-p_2}} = \frac{\frac{\left(\frac{a}{a+b}\right)}{\left(\frac{b}{a+b}\right)}}{\frac{\left(\frac{c}{c+d}\right)}{\left(\frac{d}{c+d}\right)}} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}. \quad (2)$$

If the odds ratio exceeds unity, this indicates that the odds of a success are greater in the first group than in the second group whereas if the odds ratio is less than unity the odds of a success are greater in the second group than in the first group. If the odds ratio equals unity, this indicates that the odds of a success are equal in both groups. The odds ratio is not capped.

An example of the use of the odds ratio is when participants receive treatment or do not receive treatment for a disease and they are cured or not cured. Getting cured is defined as a “success” and so the issue is whether the odds of a success are increased in the treatment condition relative to the no treatment condition. An advantage of the odds ratio over the phi coefficient is that the odds ratio is not sensitive to the margin

frequencies and so it can be used effectively even with wildly disparate margin frequencies. An advantage of the phi coefficient over the odds ratio is that the phi coefficient is more easily interpreted provided that the margin frequencies are equal (Lipsey & Wilson [4]). However, the interpretability of the odds ratio can be increased by converting it to Yule's  $Q$ , which scales the odds ratio so that it is capped at 1, as Equation (3) shows below:

$$Q = \frac{OR - 1}{OR + 1}. \quad (3)$$

A third way to characterize  $2 \times 2$  tables is to use the difference between conditional proportions ( $D$ ). The basic idea is to find the difference in row and column proportions of successes, as Equation (4) shows. Like the phi coefficient, the difference between conditional proportions is highly sensitive to having unequal margin frequencies

$$D = \frac{a}{a + b} - \frac{c}{c + d}. \quad (4)$$

## 2. The Adjusted Success Rate

It is possible to develop a fourth alternative by taking seriously the idea that for the researcher who hypothesizes that there is an association between two binary variables, a success increases the frequency of one of the cells along the major diagonal of Table 1 ( $a$  or  $d$ ) whereas a failure increases the frequency of one of the cells along the minor diagonal of Table 1 ( $b$  or  $c$ ). (It also is possible for successes to be along the minor diagonal and for failures to be along the major diagonal but I will put off discussion of this case until later.)

To make this discussion tangible, imagine that males and females are asked to indicate whether they like action movies better than romantic movies or like romantic movies better than action movies. Suppose that the researcher hypothesizes an association between sex and movie preference such that action movies should be more preferred by males and romantic movies more preferred by females. In this scenario, each case of a male preferring action movies or a female preferring romantic

movies would be a success whereas each case of a male preferring romantic movies or a female preferring action movies would be a failure. Note the difference between how a success is defined here compared to how it is defined in the odds ratio. In the odds ratio, success is defined in absolute terms, such as when a person is cured. In the present case, a success is defined in relative terms, based on the hypothesis, so that what counts as a success depends on the conjunction of appropriate levels of the two variables. For example, it is the conjunction of being male and preferring action movies or being female and preferring romantic movies that counts as a success.

Based on the foregoing discussion, there are three types of success measures that will figure into the equation to be proposed. The first success measure is the proportion of observed successes ( $S_O$ ) that is rendered as Equation (5) below:

$$S_O = \frac{a + d}{N}. \quad (5)$$

The second success measure is the expected proportion of successes due to chance. Each expected cell frequency (due to chance) is calculated the same way as in a Chi-Square test, by multiplying the appropriate row and column margin frequencies and dividing by  $N$ . For example, to find the expected frequency in cell  $a$  one multiplies the row 1 and column 1 frequencies and divides by  $N$ . Given that the expected cell frequencies have been computed, Equation (4) below can be used to obtain the expected proportion of successes, due to chance ( $S_E$ ). In Equation (6), the expected cell frequencies for cell  $a$  and cell  $d$  are designated as  $ECF_a$  and  $ECF_d$ , respectively,

$$S_E = \frac{ECF_a + ECF_d}{N}. \quad (6)$$

The third success measure is the maximum possible proportion of successes. This represents the ideal case where the researcher's hypothesis works as well as possible, given the constraints imposed by the margin frequencies. When the margin frequencies equal each other,

the maximum proportion of successes equals unity. But when the margin frequencies are unequal, this does not have to be so. For example, suppose that the researcher in the example obtained a sample of 100 males and 100 females but that 180 participants preferred action movies and only 20 participants preferred romantic movies. In this case, the maximum possible proportion of successes is .33.

To find the maximum possible proportion of successes, designated hereafter as  $S_{MAX}$ , it is necessary first to find the maximum possible success frequencies. To accomplish this, it suffices for the researcher to look at the appropriate row frequency and the appropriate column frequency: the lower of these two gives the maximum possible successes in the corresponding cell. For example, to find the maximum possible successes for cell  $a$ , one notes that the row 1 frequency is 100 (number of males) and the column 1 frequency (number of participants who prefer action movies) is 180, so that the lower of these values is 100. Thus, the maximum possible frequency for cell  $a$  is 100. Likewise, the two margin frequencies for females who prefer romantic movies are 100 and 20, and so the maximum possible frequency for cell  $d$  is 20. Equation (7) gives  $S_{MAX}$  provided that one has obtained the maximum possible frequency for cell  $a$  ( $MAX_a$ ) and the maximum possible frequency for cell  $d$  ( $MAX_d$ ).

$$S_{MAX} = \frac{MAX_a + MAX_d}{N}. \quad (7)$$

Having assembled  $S_O$ ,  $S_E$ , and  $S_{MAX}$ , it only remains to combine these to obtain the adjusted success rate ( $S_{ADJ}$ ). Equation (8) accomplishes this. There is historical precedent for Equation (8), though of an indirect nature. Roenker et al. ([5]) employed an equation similar to Equation (8) to characterize the extent to which recall protocols were clustered at a level greater than chance or not. In addition, to calculate agreement among raters, it is possible to use a Kappa coefficient that is similar to Equation (8) except that it uses 1 in the denominator rather than  $S_{MAX}$  (see Siegel & Castellan [6] for a review).

$$S_{ADJ} = \frac{(S_O - S_E)}{(S_{MAX} - S_E)}. \quad (8)$$

### 2.1. Characteristics of the adjusted success rate

What are the characteristics of the adjusted success rate? Most notably, and in contrast to the phi coefficient and the difference between conditional proportions, the adjusted success rate is capped at 1 regardless of the margin frequencies. We considered earlier the following example:  $a = 99$ ,  $b = 1$ ,  $c = 81$ , and  $d = 19$ . In this example, the maximum possible phi coefficient, given the margin frequencies, is .33 and the actual phi coefficient is .30. In contrast, the maximum possible adjusted success rate is unity, and the actual adjusted success rate is .9.

Another characteristic of the adjusted success rate is that zero represents chance successes. To see this, simply set  $S_O$  and  $S_E$  as equal to each other in Equation (6). This renders the numerator equal to zero and so  $S_{ADJ}$  similarly equals 0. Because 0 represents the case where the observed successes equal that which would be expected due to chance, and 1 represents the case where the maximum possible frequency of successes has been obtained, interpretation is rendered extremely easy, regardless of the margin frequencies. That is, for example, when the adjusted success rate is .9, it means that after adjusting for chance successes, the success rate is 90%.

### 2.2. The negative adjusted success rate

Thus far, I have assumed that the successes are along the major diagonal and the failures are along the minor diagonal. Suppose that the reverse is so. In this case, the researcher has at least two options. One option is to reverse rows or columns so that the successes are along the major diagonal. The second option, if the minor diagonal is of theoretical interest (negative relationship is of theoretical interest), is to modify the adjusted success rate to reflect that successes are along the minor diagonal. Let us explore the latter option now.

In the event that there are more successes along the minor than along the major diagonal, the floor of the adjusted success rate may not equal  $-1$ . So we need a new method if we desire a cap at  $-1$  where  $0$  continues to represent the case where the observed successes are at the chance level. To make the new method work, it is necessary to obtain the minimum possible frequency of successes in cells  $a$  and  $d$  as opposed to finding the maximum possible frequency of successes in these cells. One can accomplish this by finding the maximum possible frequency of successes in cells  $b$  and  $c$  and obtain the minimum possible frequency of successes in cells  $a$  and  $d$  by subtraction from appropriate row or column margin frequencies. Once this has been done, the minimum possible successes can be computed by using Equation (9) below:

$$S_{MIN} = \frac{MIN_a + MIN_d}{N}. \quad (9)$$

In turn, when the majority of successes are hypothesized to be along the minor diagonal, Equation (10) renders the appropriate computation of the adjusted success rate, designated as  $NS_{ADJ}$  to reflect that it is actually a negative adjusted success rate.

$$NS_{ADJ} = \frac{S_O - S_E}{S_E - S_{MIN}}. \quad (10)$$

For example, modifying the example about sex and movie preference, suppose that the hypothesis, counter to stereotypes, were that males would exhibit a relative preference for romantic movies and females would exhibit a relative preference for action movies. In addition, let us continue to keep the row margins at 100 each and continue the column split of 180 versus 20. In that case, the minimum possible frequency of successes would be 80 and 0 in cells  $a$  and  $d$ , respectively. Suppose the cell frequencies are as follows:  $a = 81$ ,  $b = 19$ ,  $c = 99$ , and  $d = 1$ . In that case, the negative adjusted success rate would equal  $-.9$ .



### 3. Comparing the Four Methods of Describing $2 \times 2$ Tables

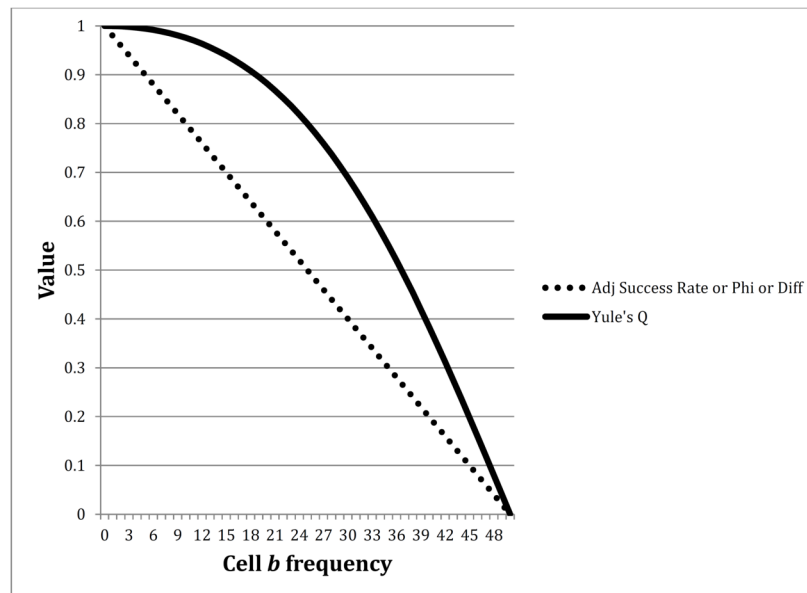
We have seen that the phi coefficient, difference between conditional proportions, and adjusted success rate are capped at 1, which enhances interpretability. The odds ratio is not capped at 1 but can be converted into Yule's  $Q$ , which is capped at 1. Therefore, from the point of view of capping at 1, there might seem to be little difference between the four methods. On the other hand, the phi coefficient and difference between conditional proportions both are highly influenced by unequal margin frequencies whereas Yule's  $Q$  and the adjusted success rate are relatively immune to unequal margin frequencies. Arguably, then, the phi coefficient and difference between conditional proportions are similar to each other, whereas Yule's  $Q$  and the adjusted success rate are similar to each other. However, this is an oversimplification and a more direct mathematical simulation is needed to investigate the similarities and differences among the four methods.

I performed three sets of mathematical simulations and the findings are illustrated via Figures 1, 2, and 3, respectively. In the first set of simulations, I set all of the margin frequencies at 100 and let the cell  $b$  frequency range from 0 (maximum successes) to 50 (50% successes). Because there is only a single degree of freedom in a  $2 \times 2$  table, given fixed margin frequencies, it follows that the frequency in one of the cells can be used to represent the whole table of data and it was convenient to use cell  $b$ . In Figure 1, the cell  $b$  frequency is presented along the horizontal axis. The idea was to determine how the phi coefficient, Yule's  $Q$ , the difference between conditional proportions, and the adjusted success rate were influenced by the cell  $b$  frequency. As Figure 1 shows, under equal margin frequencies, the phi coefficient, the difference between conditional proportions, and the adjusted success rate all equal each other with Yule's  $Q$  being the odd method out. It is possible to argue that this is a strike against Yule's  $Q$ . A possible second strike against Yule's  $Q$  is that its decrease as the cell  $b$  frequency decreases is not

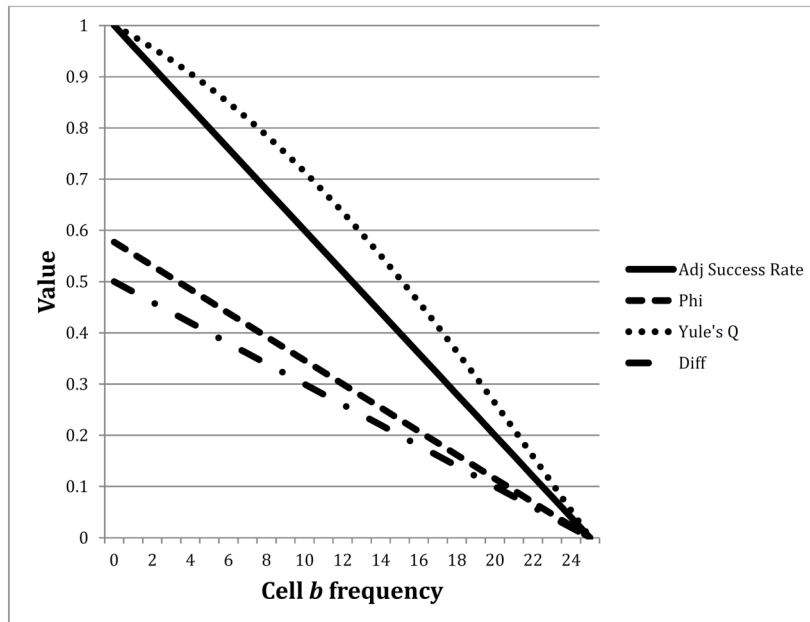
linear. A third strike, though perhaps a trivial one, against Yule's  $Q$  is that it is not possible to calculate it when the cell  $b$  (or cell  $c$ ) frequency is zero because the odds ratio that provides the basis for the calculation is undefined. For ease of viewing, I set Yule's  $Q$  at 1 in the case where the cell  $b$  frequency is 0, in Figures 1, 2, and 3 but this is not strictly correct.

In the second set of mathematical simulations, I set the row frequencies at 100 each but had mildly unequal column frequencies of 150 and 50 for *column 1* and *column 2*, respectively. For this set of simulations, I let the cell  $b$  frequency vary from 0 (maximum possible number of successes given the margin frequencies) to 25 (50% successes in *column 2*). As in Figure 1, the values of the phi coefficient, Yule's  $Q$ , the difference between conditional proportions, and the adjusted success rate are expressed as a function of the cell  $b$  frequency. In contrast to Figure 1, Figure 2 shows that all four of the methods render quite different curves. The phi coefficient and difference between conditional proportions fall well short of 1 even when there are the maximum possible successes. In this respect, Yule's  $Q$  and the adjusted success rate are superior. In addition, consistent with Figure 1, the decrease in Yule's  $Q$  as the cell  $b$  frequency decreases is nonlinear whereas it is linear for the other three methods.

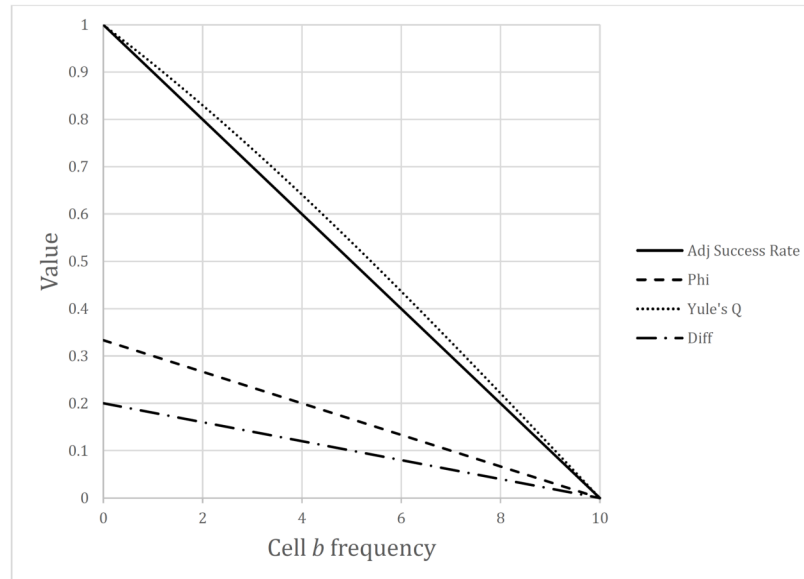
In the third set of mathematical simulations, I again set the row frequencies at 100 each but had extremely unequal column frequencies of 180 and 20 for *column 1* and *column 2*, respectively. Thus, the cell  $b$  frequency varied from 0 (maximum possible number of successes given the margin frequencies) to 10 (50% successes in *column 2*). In Figure 3, the phi coefficient and difference between conditional proportions both fall extremely far from 1, even when successes are maximized, with the difference between conditional proportions performing the worst in this respect. Yule's  $Q$  and the adjusted success rate follow quite similar decreases as the cell  $b$  frequency decreases, though the decrease of Yule's  $Q$  is nonlinear whereas the decrease of the adjusted success rate is linear.



**Figure 1.** The phi coefficient, Yule's  $Q$ , the difference between conditional proportions, and the adjusted success rate vary with the cell  $b$  frequency. Put more conceptually, the phi coefficient and adjusted success rate vary from the case where the proportion of observed successes is at the maximum possible to where the proportion of observed successes is at the chance level. Although Yule's  $Q$  is undefined when the cell  $b$  frequency is zero, it is set at 1 to render the figure easier to see. In Figure 1, the margin frequencies are equal.



**Figure 2.** The phi coefficient, Yule's  $Q$ , the difference between conditional proportions, and the adjusted success rate vary with the cell  $b$  frequency. Although Yule's  $Q$  is undefined when the cell  $b$  frequency is zero, it is set at 1 to render the figure easier to see. In Figure 2, the row frequencies are equal but the column frequencies are unequal (150 and 50 for column 1 and column 2, respectively).



**Figure 3.** The phi coefficient, Yule's  $Q$ , the difference between conditional proportions, and the adjusted success rate vary with the cell  $b$  frequency. Although Yule's  $Q$  is undefined when the cell  $b$  frequency is zero, it is set at 1 to render the figure easier to see. In Figure 3, the row frequencies are equal but the column frequencies are extremely unequal (180 and 20 for column 1 and column 2, respectively).

In summary, when the margin frequencies are equal, Yule's  $Q$  is quite different from the other three methods both in terms of being nonlinear and also giving quite different values. When the column frequencies are mildly unequal, all four methods are different from each other with the adjusted success rate being the only method that is both linear and gives a value of 1 when successes are maximized. And when the column frequencies are extremely unequal, the adjusted success rate again is the only method that is both linear and gives a value of 1 when successes are maximized, though Yule's  $Q$  is reasonably close to the adjusted success rate at most cell  $b$  frequencies. If one believes that giving a value of 1 when successes are maximized, regardless of the margin frequencies is an advantage; and also that linearity is an advantage; then the adjusted success rate stands out as the single method

that has all of the advantages. Or to put it negatively, if sensitivity to margin frequencies and nonlinearity are considered to be disadvantages, only the adjusted success rate is devoid of these disadvantages. But is the adjusted success rate limited to the case of the  $2 \times 2$  table?

#### **4. Applying the Adjusted Success Rate to a $2 \times 2 \times 2$ Frequency Table**

Suppose that one has a  $2 \times 2 \times 2$  frequency table (e.g., see Table 2). It is not clear immediately how to apply the phi coefficient, difference between conditional proportions, or odds ratio to obtain an intelligible interpretation. In contrast, the adjusted success rate can be applied in a straightforward manner. To see that this is so, consider an example from the reasoned action approach to the causation of behaviour (Ajzen & Fishbein [1]; Fishbein & Ajzen [2]; Fishbein & Ajzen [3]). If we restrict ourselves to the attitudinal part of the theory, the straightforward hypothesis is that attitudes cause behavioural intentions, which, in turn, cause behaviours. With this in mind, suppose that a researcher obtains binary measures of the three variables. Thus, attitudes are scored as being against the behaviour or for it  $\{0, 1\}$ , participants intend not to perform the behaviour or perform it  $\{0, 1\}$ , and people actually do not perform the behaviour or perform it  $\{0, 1\}$ . Well, then, there are eight possible sequences of scores and only Sequence 1  $\{0 - 0 - 0\}$  and Sequence 8  $\{1 - 1 - 1\}$  are consistent with the hypothesis.

- (1)  $0 - 0 - 0$  (cell *a* in Table 2)
- (2)  $0 - 0 - 1$  (cell *b* in Table 2)
- (3)  $0 - 1 - 0$  (cell *c* in Table 2)
- (4)  $0 - 1 - 1$  (cell *d* in Table 2)
- (5)  $1 - 0 - 0$  (cell *e* in Table 2)
- (6)  $1 - 0 - 1$  (cell *f* in Table 2)
- (7)  $1 - 1 - 0$  (cell *g* in Table 2)
- (8)  $1 - 1 - 1$  (cell *h* in Table 2)

**Table 2.** The production of cell frequencies in a  $2 \times 2 \times 2$  table with binary variables. The margin frequencies are as follows: Level 1 of Variable 1 =  $a + b + e + f$ , Level 2 of Variable 1 =  $c + d + g + h$ , Level 1 of Variable 2 =  $a + c + e + g$ , Level 2 of Variable 2 =  $b + d + f + h$ , Level 1 of Variable 3 =  $a + b + c + d$ , and Level 2 of Variable 3 =  $e + f + g + h$

		Variable 3			
		0		1	
Variable 1		Variable 1		Variable 1	
		0	1	0	1
0		$a$	$b$	$e$	$f$
1		$c$	$d$	$g$	$h$

To apply the adjusted success rate to the  $2 \times 2 \times 2$  table, one performs the same steps as in the  $2 \times 2$  case. That is, it is necessary to obtain the observed proportion of successes ( $S_O$ ), the expected proportion of successes ( $S_E$ ), and the maximum possible proportion of successes given the limits imposed by the margin frequencies ( $S_{MAX}$ ). Once these are obtained, Equation (8) applies, as usual. To aid in illustration, consider Table 2, where cells  $a$  and  $h$  are the ones that indicate success  $\{0 - 0 - 0\}$  and  $\{1 - 1 - 1\}$ , respectively.

The total number of participants is the sum across all cells ( $N = a + b + c + d + e + f + g + h$ ). Thus, Equation (11) renders the observed proportion of successes.

$$S_O = \frac{a + h}{N}. \quad (11)$$

To obtain the expected proportion of successes, it is useful to find the expected cell frequencies first. One way to do this is to find the product of the three relevant margin frequencies and divide by  $N^2$ . For example, to find the expected cell  $a$  frequency ( $ECF_a$ ), the researcher could obtain the product of the Level 1 frequencies of the three variables and divide

that product by  $N^2$ . To find the expected cell  $h$  frequency ( $ECF_h$ ), the researcher could obtain the product of the Level 2 frequencies of the three variables and divide that product by  $N^2$ . Once this has been done, Equation (12) renders the expected proportion of successes.

$$S_E = \frac{ECF_a + ECF_h}{N}. \quad (12)$$

To obtain the maximum possible proportion of successes, it is useful to obtain the maximum possible frequency of successes in cells  $a$  ( $MAX_a$ ) and  $h$  ( $MAX_h$ ). Once these are obtained by finding the smallest relevant margin frequency, Equation (13) renders the maximum possible proportion of successes, given the limits imposed by the margin frequencies.

$$S_{MAX} = \frac{MAX_a + MAX_h}{N}. \quad (13)$$

Just as in the case of  $2 \times 2$  tables, once the researcher has assembled  $S_O$ ,  $S_E$ , and  $S_{MAX}$ , these can be combined to obtain the adjusted success rate ( $S_{ADJ}$ ) via instantiation into Equation (8), as we already have seen.

Let us reconsider the example of the hypothesis involving attitudes, behavioural intentions, and behaviours. Suppose that the margin frequencies are as follows: Level 1 of attitudes = 100, Level 2 of attitudes = 100, Level 1 of behavioural intentions = 100, Level 2 of behavioural intentions = 100, Level 1 of behaviours = 180, and level 2 of behaviours = 20. Thus, in the case of behaviours, the margin frequencies are wildly unequal. With these margin frequencies, the expected cell frequencies are 45 and 5 for cells  $a$  and  $h$ , respectively. The maximum possible cell frequencies are 100 and 20 for cells  $a$  and  $h$ , respectively. Thus,  $S_E = \frac{45 + 5}{200} = .25$  and  $S_{MAX} = \frac{100 + 20}{200} = .6$ . Also suppose that the actual cell frequencies are 95 and 15 for cells  $a$  and  $h$ , respectively, so



that  $S_O = \frac{95 + 15}{200} = .55$ . Instantiating these values into Equation (8)

gives the following solution for the adjusted success rate:

$$S_{ADJ} = \frac{.55 - .25}{.6 - .25} = .857.$$

## 5. Conclusion

The suggestion that researchers use the adjusted success rate to describe  $2 \times 2$  tables should not be taken as an argument that phi coefficients, differences between conditional proportions, and odds ratios should be eliminated. There are times when it is important to have an index that provides the actual correlation coefficient and the phi coefficient is useful for this purpose. Potential performance theory (Trafimow & Rice [7]) constitutes an example that makes use of the phi coefficient—not merely as a description of a  $2 \times 2$  table but literally as a correlation coefficient. Or consider if one were to make a decision about whether to undergo a particular treatment for a disease. In this case, knowledge about how much more likely a positive outcome would be in the case of treatment relative to the case where there is no treatment would be desirable, and the odds ratio provides that information. As indicated earlier, when a “success” is defined in absolute terms, such as being cured of a disease, the odds ratio is a good thing to have.

But sometimes the goal is to describe how much the  $2 \times 2$  table (or more complex table) supports a hypothesis. In this case, the definition of a “success” is not absolute but relative to the hypothesis. We saw an example where males who prefer action movies or females who prefer romantic movies were considered to be successes but where a particular movie preference, considered in isolation, could not be considered to be a success or failure. This contrasts with the example of cure for a disease, where cure is always a success regardless of the presence or absence of treatment. When the goal is to determine how well the data support the hypothesis, the adjusted success rate has the advantages of being directly interpretable, of being insensitive to unequal margin frequencies, and of linearity.

It is interesting to consider the issue of disease and cure from an additional perspective. The relevance of the odds ratio is obvious but from the point of view of a scientist, who wishes to know how strong the support is for the hypothesis that links treatment to cure, the adjusted success rate provides valuable information about the proportion of successes while taking chance into account. The odds ratio, even when converted to Yule's  $Q$ , is less intelligible for this purpose. Furthermore, if the margin frequencies differ, the phi coefficient and difference between conditional proportions also are difficult to interpret.

Another potential advantage of the adjusted success rate is that it aids in comparing effects across studies. Suppose that there are two experiments and each one results in an odds ratio for a different treatment, relative to a no treatment control group. Although the treatment with the larger odds ratio might be preferred to the treatment with the smaller odds ratio, it is not immediately obvious how to compare them to each other, at least not without some extra mathematical manipulation. The problem is exacerbated if one attempts to compare phi coefficients or difference between conditional proportions, especially if the distributions of margin frequencies differ across the two experiments. In contrast, the difference between adjusted success rates renders the two treatments comparable on an immediate basis, without any further calculations, and regardless of differences in the distribution of margin frequencies across the two experiments.

Finally, the adjusted success rate can extend to larger tables. The reasoning that already has been applied to  $2 \times 2$  tables and to  $2 \times 2 \times 2$  tables can be extended indefinitely.

In conclusion, the adjusted success rate is an additional way to describe the association between binary variables. Although the phi coefficient, difference between conditional proportions, and the odds ratio are excellent for particular purposes, none of these is ideal for understanding the empirical success of the hypothesis. The adjusted success rate remedies this lack.

**References**

- [1] I. Ajzen and M. Fishbein, *Understanding Attitudes and Predicting Social Behavior*, Prentice-Hall, Englewood Cliffs, NJ, 1980.
- [2] M. Fishbein and I. Ajzen, *Belief, Attitude, Intention and Behavior: An Introduction to Theory and Research*, Addison-Wesley, Reading, MA, 1975.
- [3] M. Fishbein and I. Ajzen, *Predicting and Changing Behavior: The Reasoned Action Approach*, Psychology Press (Taylor & Francis), New York, 2010.
- [4] M. W. Lipsey and D. B. Wilson, *Practical Meta-Analysis*, Sage Publications, London, 2001.
- [5] D. L. Roenker, C. P. Thompson and S. C. Brown, Comparison of measures for the estimation of clustering in free recall, *Psychological Bulletin* 76(1) (1971), 45-48.  
DOI: <http://dx.doi.org/10.1037/h0031355>
- [6] S. Siegel and N. J. Castellan, *Nonparametric Statistics for the Behavioral Sciences* (2nd Edition), McGraw-Hill, Inc., New York, 1988.
- [7] D. Trafimow and S. Rice, Potential performance theory (PPT): A general theory of task performance applied to morality, *Psychological Review* 115(2) (2008), 447-462.  
DOI: <https://doi.org/10.1037/0033-295X.115.2.447>

