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ROLE OF SKEW-SYMMETRIC DIFFERENTIAL FORMS IN FIELD THEORY. FOUNDATIONS OF THE UNIFIED AND GENERAL FIELD THEORIES

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Abstract

Role of skew-symmetric forms in field theory is explained by the fact that they correspond to the conservation laws.

It was shown that the solutions to the field theory equations (such as the equations by Dirac, Schr\H{o}dinger, Maxwell, Einstein and so on) are closed exterior skew-symmetric differential forms corresponded to conservation laws for physical fields. In this case, the degree of closed exterior form is a parameter that integrates field theories in unified theory.

Then it was shown that from the mathematical physics equations, which consist of the equations of conservation laws for material media and describe material media, it follows the evolutionary relation in skew-symmetric differential forms that possesses the properties of field theory equations. The evolutionary relation is a non-identical relation for functionals such as the action functional, entropy, Pointing's vector, Einstein's tensor, wave function, and others. As it is known, the field theory equations are equations for such functionals. This points out to

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a correspondence between the evolutionary relation mathematical physics equations that lies at the basis of the general field theory. And the field theory equations and discloses a connection between the field theory equations and the equations of mathematical physics.

The paper aims to show that the field theory equations, which describe physical fields, follow from the equations of mathematical physics, which describe material media. This is based on the properties of conservation laws that lie at the basis of unified and general field theories. (The conservation laws for physical fields and the conservation laws for material media are different conservation laws.)

1. Introduction

In present paper, the accent is made on the basic principles that disclose foundations of united and general field theories.

The results of present paper are based on the properties of conservation laws. They are obtained with the help of skew-symmetric differential forms which properties correspond to conservation laws. (In doing so the mathematical formalism which was disclosed by the author and possesses nontraditional elements was applied.)

The conservation laws for physical fields mean the existence of conservative quantities or objects. Such conservation laws are described by the closed exterior differential skew-symmetric forms that are conservative quantities since the closed form differential equals zero. In Section 1, it is shown that the closed exterior forms are solutions to the field theory equations. In this case, the degree of closed exterior forms is a parameter that integrates field theories into the unified theory.

It turns out that closed exterior forms, which lies at the basis of field theories, are generated by skew-symmetric forms that corresponds to conservation laws for material media (material systems such as the thermodynamic, gas-dynamic and cosmic systems, as well as the systems of charged particles and others). These skew-symmetric forms are obtained from the mathematical physics equations (which consist of the equations of conservation laws for material media). This points out to correlation between the field theory equations and the mathematical physics equations. This correlation lies at the basis of general field theory. (The conservation laws for physical fields (described by the field-theory equations) and the conservation laws for material media (described by the equations of mathematical physics) are different conservation laws.)

These results follow from unique hidden properties of the mathematical physics equations that were found (discovered) by present author. From the equations of mathematical physics, it follows **the evolutionary relation for state functionals such as** the action functional, entropy, Pointing's vector, Einstein's tensor, wave function, and others, that are also **functionals of field theory.** This relation, firstly, discloses the evolutionary properties of the mathematical physics equations that enables one to describe the origination of various physical structures and formations, and, secondly, possesses the properties of field theory equations.

The evolutionary relation discloses an internal connection of the field theory equations and a connection between the field theory equations and the mathematical physics equations. This points to the fact that, firstly, equations describing physical fields are not independent ones, and, secondly, physical fields are generated by material media. That is, material media and physical fields make a whole.

One can read about the properties of skew-symmertic forms in the papers [1, 2] and in paper [3]: http://arxiv.org/abs/1007.4757.

2. Role of Closed Exterior Forms in Field Theory

Role of closed exterior forms in field theory is explained by the fact that they describe the conservation laws for physical fields, namely, to existence of a conservative quantities or objects (structures).

2.1. Some properties of closed exterior skew-symmetric forms corresponding to the conservation laws [3]: http://arxiv.org/abs/1007.4757

From the closure conditions for exterior differential form

$$d\theta^k = 0, \tag{1}$$

one can see that the closed exterior differential form is a conservative quantity (θ^k is the exterior differential form of degree k (k-form)). This means that it can correspond to a conservation law, namely, to existence of a conservative physical quantity. From the closure conditions for inexact exterior differential form, i.e., that is closed only on pseudostructure, follows that

$$d_\pi \theta^k = 0, \tag{2}$$

$$d_\pi {}^*\theta^k = 0, \tag{3}$$

where ${}^*\theta^k$ is a dual form that describes the pseudostructure π. From conditions (2) and (3), one can see that the closed exterior form and the dual form constitute a conservative object, namely, the pseudostructure (the dual form) with conservative quantity (closed inexact form). That is, a closed inexact exterior form and relevant dual form describe a physical structure, namely, a structure on which a conservation law fulfills.

The conservation laws for physical fields are such conservation laws. In this case, closed inexact exterior and dual forms describe a physical structure, namely, a structure with conservative quantity, namely, a structure on which conservation laws for physical fields fulfill. The physical structures from which physical fields are formatted may be such structures.

This substantiates the fact that the properties of closed inexact forms and relevant dual forms must lie at the basis of theories that describe physical fields.

2.2. Closed exterior forms as solutions to the field theory equations

The closed exterior or dual forms are solutions to the field-theory equations. In this case the solutions to quantum mechanics equations are closed exterior forms of zero degree, ones to equations of Hamilton formalism are forms of first degree, and the solutions to electromagnetic field equations are forms of second degree. (It should be noted that the field theory equations have non-unique solution.)

One can see that the field theory equations are connected with closed exterior forms of a certain degree. This allows for introducing a classification of physical fields and interactions in degrees of closed exterior forms. *It turns out that the degree of closed exterior forms is a parameter that integrates field theory equations into a unified field theory*. The closed exterior forms disclose the properties that are common for field theory equations. This is a step to building a unified field theory.

Below it will be shown that the closed inexact exterior forms, which describe conservation laws for physical fields and are solutions to field theory equations, are generated by skew-symmetric forms that are obtained from the mathematical physics equations, which describe conservation laws for material media. This will point out to a connection between the field theory equations and the mathematical physics equations that lies at the basis of the general field theory.

3. Hidden Properties of the Mathematical Physics Equations Describing Material Media

As it is known, the mathematical physics equations, which describe material media (material systems) such as the thermodynamic, gas-dynamic and cosmic systems, as well as the systems of charged particles and others, are composed of the conservation law equations for energy, linear momentum, angular momentum, and mass that are conservation laws for material media. (Examples of the mathematical physics equations and references to such equations are presented in Appendix 1.) [Here it should clue attention to the fact that the conservation laws for material media (as opposed to conservation laws for physical fields) are differential ones. The skew-symmetric forms correspond to such conservation laws.]

It turns out that the equations of mathematical physics possess ***hidden properties*** that doesn't follow directly from the mathematical physics equations. Such properties, which are conditioned by conservation laws, are discovered under additional condition of a consistence of conservation law equations that made up the equations of mathematical physics. In this analysis the evolutionary relation possessing unique possibilities is obtained. This relation discloses the evolutionary properties of mathematical physics equations describing material media and a connection of field theory equations and the equations of mathematical physics. (As it will be shown below, this relation possesses the properties of field theory equations.)

The evolutionary relation is obtained when studying a consistency of the conservation laws equations in the mathematical physics equations.

3.1. Analysis of a consistency of conservation laws equations: Evolutionary relation for state functionals

The evolutionary relation is a relation for functionals describing a state of material medium, that is, they are state functionals. It turns out that functionals such as wave function, entropy, the action functional, the Pointing vector, the Einstein tensor and so on, which are the field theory functionals, are also functionals describing a material medium state [4].

For investigation of the consistency of the conservation law equations it is necessary to use two nonequivalent frames of reference, namely, the inertial frame of reference (the Euler frame of reference is an example of such frame) and the accompanying frame of reference that is connected with the manifold made up by the trajectories of material system elements (the Lagrange frame of reference is an example of such a frame).

Let us analyze the correlation of the equations that describe the conservation laws for energy and linear momentum. In the inertial frame of reference the energy equation can be reduced to the following form [4]:

$$\frac{D\psi}{Dt} = A_1. \tag{4}$$

Here ψ is the state functional, A_1 is a quantity that depends on specific features of material system (material medium) and on external energy actions onto the system.

In the accompanying frame of reference the total derivative with respect to time is a derivative along the trajectory. For this reason, in the accompanying frame of reference, the equation of energy is written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1, \tag{5}$$

here ξ^1 are the coordinates along the trajectory.

In the accompanying frame of reference, the equation for linear momentum also reduced to the equation for state functional:

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu,\ \nu = 2, \ldots, \tag{6}$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of material system and force actions.

Here a certain peculiarity rises, namely, there are two equations for the functional ψ. Such a peculiarity lust leads to nontraditional effects.

Since Equations (5) and (6) are expressions for derivatives along different directions, they can be convoluted into the relation

$$d\psi = A_\mu d\xi^\mu, \quad \mu = 1, \nu. \tag{7}$$

Relation (7) can be rewritten as

$$d\psi = \omega, \tag{8}$$

here $\omega = A_\mu d\xi^\mu$ is the skew-symmetric differential form of the first degree. (A summing over repeated indices is carried out.)

When studying a consistence of equations for energy, linear momentum, angular momentum and mass the relation (8) will be the written as

$$d\psi = \omega^p, \tag{9}$$

here ω^p is the form degree p (p takes the values $p = 0, 1, 2, 3$).

Since the conservation laws equations are evolutionary ones, the relations obtained are also evolutionary relations, and the skew-symmetric forms ψ and ω^p are evolutionary ones. [A concrete form of relation (7) is presented in papers [1, 2]. In the paper http://arxiv.org/pdf/math-ph/0310050v1.pdf relation (9) for $p = 2$ was considered for electromagnetic field. In this case the functional ψ is the Pointing's vector. The relation for Einstein's tensor is obtained when integrating the evolutionary relation for $p = 3$.]

3.2. Correspondence between evolutionary relation and the field theory equations

It has been shown that from the mathematical physics equations it follows the evolutionary relation which is a relation for functionals such as the action functional, entropy, wave function, Lagrangian, Einstein's tensor, Pointing's vector and others. As it is known, the field theory equations are equations for such functionals. This points out to a correspondence between the evolutionary relation and the field theory equations.

Further it will be presented a substantiation of such a correspondence.

As shown, solutions to field theory equations are closed exterior forms that describe physical structures on which conservation laws for physical fields are fulfilled and of which physical fields are formatted.

It will be shown that closed exterior forms, which describe such physical structures, are realized from the evolutionary relation obtained from the mathematical physics equations that describe material media. (This points out to a connection of the field theory equations with the mathematical physics equations. This is a proof of the fact that physical structures that made up physical fields are generated by material media.)

3.3. Peculiarities of evolutionary relation

The evolutionary relation obtained from the conservation laws equations possesses nontraditional peculiarities, namely, appears non-identical and self varying one. Such peculiarities allow for discover hidden properties of mathematical physics equations, namely, the presence of ***double solutions*** and ***a duality of functional*** that allows for description of evolutionary processes and origination of physical structures and observable formations.

Non-identity of evolutionary relation. The evolutionary relation consists of skew-symmetric differential forms. In this case, in the right-hand side it stands the skew-symmetric differential form that, as opposed to external form (with the basis defined on integrable manifolds or structures) is defined on ***non-integrable manifold.*** This follows from the derivation of the evolutionary relation. The evolutionary relation was obtained in the accompanying frame of reference, which is connected with the manifold built up by the trajectories of the material system elements. Such a manifold is a deforming non-integrable one. It turns out that the skew-symmetric form in evolutionary relation is defined on non-integrable manifold manifold [3]: http://arxiv.org/abs/1007.4757.

Such skew-symmetric form that appears to be unclosed and evolutionary one possesses special properties because of which in evolutionary relation it will appear the non-traditional peculiarities which have a unique physical meaning.

The skew-symmetric form defined on non-integrable manifold cannot be closed since the differential of this form is nonzero. As opposed to the exterior form differential, an additional term, namely, the differential of the basis (that is a non-integrable manifold) will appear in the evolutionary form differential [3, 5]. This is a differential of the metric form of manifold. Such differential is nonzero since the manifold is non-integrable one. As the result, the differential of evolutionary skew-symmetric form cannot be equal to zero. And for this reason, the evolutionary form, as opposed to exterior one, cannot be closed, that is, it cannot be an invariant.

[For example, the skew-symmetric form $\omega = A_\mu d\xi^\mu$ in relation (7) is not a closed form since its differential is nonzero. The differential $d\omega$ of the form $\omega = A_\mu d\xi^\mu$ can be written as $d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$, where (see [3]):

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta}\right) + \left(\Gamma^\sigma{}_{\beta\alpha} - \Gamma^\sigma{}_{\alpha\beta}\right) A_\sigma.$$

The first term in the commutate or of the form ω is nonzero since the coefficients A_μ of the form ω depend on different actions (energetic and force) and for this reason their derivatives appear to be inconsistent. The expressions $(\Gamma^\sigma{}_{\beta\alpha} - \Gamma^\sigma{}_{\alpha\beta})$ entered into the second term are just components of the metric form commutate or that specifies the manifold deformation and hence is nonzero [3]. (About the properties of the metric form commutators of non-integrable manifolds one can read in the paper [6].) Thus, it results that the commutate or, and hence a differential of the form ω, are nonzero, and the form ω proves to be unclosed and cannot be a differential. (Below, the physical meaning of the commutate or of evolutionary skew-symmetric form will be disclosed.)]

It turns out that the evolutionary skew-symmetric form entered into evolutionary relation whose basis is non-integrable manifold, and hence it cannot be closed and be an differential (an invariant), i.e., a measurable quantity. And this means that ***the evolutionary relation to be non-identical one*** since it includes a non-measurable term.

The nonidentity of the evolutionary relation is connected with the properties of conservation laws for material media. From the non-identity of evolutionary relation it follows that the equations of conservation laws turn out to be inconsistent. And this points to a non-commutativity of conservation laws. (As it will be shown below, the non-commutativity of conservation laws is a moving force of evolutionary processes accompanied by emergence of various structures.)

Self-variation of evolutionary relation. It should be noted one more property of evolutionary relation, namely, e***volutionary relation proves to be a self-varying relation***, because, firstly, it is an evolutionary relation, and, secondly, it contains two objects that are incommensurable.

The non-identical evolutionary relation obtained from the mathematical physics equations possesses unique properties. As it will be shown below, the ***closed*** exterior forms, which describe physical structures, are obtained from the skew-symmetric ***unclosed*** form entered into this relation and are solutions to equations of mathematical physics. Such closed exterior forms describe physical structures, which made up physical fields.

(Moreover, it will be shown that the evolutionary relation also clarifies ***the process of emergence of physical structures*** that made up physical fields.)

Realization of physical structures. In the paper [3]: http://arxiv.org/abs/1007.4757, it was shown that from the skew-symmetric form defined on non-integrable manifold the closed inexact exterior form is obtained under degenerate transformation. The skew-symmetric evolutionary form entered into evolutionary relation possesses such a property.

It turns out that, if some degrees of freedom arises when describing material media, it is realized the degenerate transformation (which doesn't conserve a differential) under which from the unclosed skew symmetric form (which differential is nonzero) the closed exterior form (with the differential being zero) will be obtained. ***The vanishing of such functional expressions as determinants, Jacobians, Poisson's brackets, residues, and others corresponds to these additional conditions***. The conditions of degenerate transformation can be realized (spontaneously) under a change of non-identical evolutionary relation that, as it was noted, appears to be a self-varying relation.

The realization of the condition of degenerate transformation means that it is realized the closed dual form ${}^*\omega^p$ that describes some structure π (this is a pseudostructure with respect to its metric properties), and the closed inexact (only on pseudostructure) exterior form ω_π^p.

The realization of a closed inexact exterior form (conservative quantity) and relevant dual form (integral structure) describe an emergence of physical structure on which exact conservation laws are obeyed. That is, the realization of closed exterior form and relevant dual form describe an emergence of physical structure on which conservation laws for physical fields are obeyed. The physical structures from which physical fields are formatted are such structures. And they are generated by material media.

In other words, from the evolutionary relations, as well as from the field theory equations, the closed exterior forms corresponding to conservation laws for physical fields are obtained. This points out to a correspondence of the evolutionary relation to the field theory equations and a connection between the field theory equations and the equations of mathematical physics.

Below it will be shown that the mathematical physics equations also discloses a mechanism of emergence of physical structures which made up physical fields. This appears to be possible due to hidden properties of the mathematical physics equations that were discovered with the help of evolutionary relation.

3.4. Hidden properties of solutions to the mathematical physics equations: Double solutions to the equations of mathematical physics

From the evolutionary relation it follows that the mathematical physics equations possess ***double solutions***. (Below it will be shown that this fact has a physical meaning since it enables one to describe the origination of various structures and observable formations.)

As it was shown, the right-hand side of evolutionary relation $d\psi = \omega^p$ contains skew-symmetric form that is not a differential. For this reason the evolutionary relation cannot be integrated directly. This means that the equations of mathematical physics, from which the evolutionary relation was obtained, prove to be non-integrable. This is, ***solutions to the original equations of mathematical physics are not functions*** (their derivatives do not made up a differential). These solutions will dependon the commutator of the evolutionary skew-symmetric form ω^p, which is nonzero.

However, the mathematical physics equations can have solutions that are discrete functions. This is possible in the case when an identical relation is obtained from a non-identical evolutionary relation.

As it was shown above, the closed exterior form is obtained from unclosed skew-symmetric form under degenerate transformation. In this case, from the evolutionary non-identical relation it is possible to obtain the identical relation composed of differentials that can be integrated. The realization of closed inexact exterior form ω_π^p leads to the fact that on pseudostructure from the evolutionary non-identical relation (9) it follows the identical relation:

$$d\psi_\pi = \omega_\pi^p, \tag{10}$$

since the form ω_π^p is a closed form, i.e., this form is a differential.

Because the identical relation consists only of differentials, this relation can be integrated. This means that ***the equations of mathematical physics become locally integrable*** **(*only on pseudostructure*)**. In this case the pseudostructure is an integrable structure. ***The solutions to the mathematical physics equations on integrable structures are generalized solutions that are discrete functions***.

3.5. The process of origin physical structures of which physical fields are made up and emergence of observable formations in material media

It was shown that the equations of mathematical physics possess double solutions, namely, solutions on non-integrable initial manifold and solutions on integral structures. And this has a physical meaning. From evolutionary relation it follows that inexact solution describes non-equilibrium state of material media, whereas exact solution describes locally-equilibrium state. (The transition from inexact solution to exact one describes the process of various structures emergence.) This is connected with the property of the functional ψ and discloses physical meaning of functionals of mathematical physics equations and field theory equations.

Non-equilibrium state of material media. The functional ψ in the left-hand side of the evolutionary relation $d\psi = \omega^p$ specifies a state of material media.

The availability of the differential of functional ψ means that there exists a state function, and this points out to an equilibrium state of material medium. However, from a non-identical relation one cannot get the differential $d\psi$. And this means т that the state of material medium is non-equilibrium one. In other words, an internal force acts in material medium. It is evident that the internal force originates at the expense of some quantity described by the evolutionary form commutator [3]. (Nonequilibrium state of material medium is caused by the properties of conservation laws, namely, by its non-commutativity. From the non-identity of the evolutionary relation it follows, as it was already noted, that the equations of conservation laws turn out to be inconsistent. And this points to a noncommutativity of conservation laws for material media.)

Transition of material medium into locally-equilibrium state: Origination of physical structures and advent of observable formations

As it was shown, under degenerate transformation caused by any degree of freedom closed inexact exterior and dual forms are obtained from evolutionary relation.

This, firstly, points out to the realization of structures on which it is obeyed the exact conservation law, namely, conservation law corresponding to physical fields. That is, this points to the realization of physical structures of which physical fields can be formatted. (Massless particles, structures made up by eikonal surfaces and wave fronts, these and so on are examples of physical structures.)

And, secondly, the realization of closed inexact exterior forms points to the fact that the identical relation is obtained. From the identical relation $d\psi_\pi = \omega_\pi^p$, one can obtain the differential of the state functional $d\psi_\pi$, and this will point to a presence of state function and a transition of material medium from non-equilibrium state into equilibrium one. In this case, unmeasurable quantity, which is described by the evolutionary form commutator and act as internal force, converts into a measurable quantity of material medium. In material medium this reveals in emergence of some observed formations (waves, vortices, fluctuations, and turbulent pulsations [7].) Such emerged formations are described by generalized solutions to the equations of mathematical physics.

Thus, it turns out that the emergence of physical structures which made up physical fields and the advent of observable formation in material media are a manifestation of the same phenomena. (The light is an example of such a duality, namely, as a massless particle (photon) and as a wave.) However, physical structures and observable formations are not identical objects. Whereas the wave is an observable formation of material medium, the element of wave front made up a physical structure (of physical field) in the process of its evolution.

4. Evolutionary Relation as the Basis of General Field Theory

It has been shown that the mathematical physics equations, which describe material media, possess unique properties. From these equations, it follows the evolutionary relation that possesses the properties of field theory equations.

The evolutionary relation is a relation for functionals such as the action functional, entropy, wave function, Lagrangian, Einstein's tensor, Pointing's vector and others. As it is known, the field theory equations are equations for such functional. This points out to a correspondence between evolutionary relation and the field theory equations.

Closed exterior forms corresponding to conservation laws for physical fields and being solutions to the field theory equations are obtained from the evolutionary relation. This substantiates a correspondence between evolutionary relation and the field theory equations.

The evolutionary relation discloses foundations of general field theory.

Correspondence between the evolutionary relation obtained from the mathematical physics equations and the field-theory equations points to the fact that the connection of the field theory equations with the mathematical physics equations describing material media lies at the basis of general field theory.

The evolutionary relation units the field theory equations, discloses the properties of the field theory equations, which are common for all equations of field theory, and internal connection of field theory equations.

From the evolutionary relation it follows that the physical structures, which made up physical fields, are physical structures that are generated by material media and are described by the equations of mathematical physics. (In more details this is written in papers [8, 9]. In present paper, the accent is made on the basic principles that disclose foundations of unified and general field theories.)

5. Conclusion

It has been shown that from the mathematical physics equations, which describe material media, it follows the evolutionary relation for functionals such as the action functional, entropy, wave function, Lagrangian, Einstein's tensor, Pointing's vector and others, which are functionals of the field theory equations. This points to a connection of the field theory equations with the equations of mathematical physics.

The evolutionary relation possesses unique properties. This relation corresponds to field theory equations. From the evolutionary relation one can obtain closed exterior forms being solutions to field theory equations and describe physical structures that made up physical fields. In this case the evolutionary relation also describes ***the process of generating physical structures by material media***. That is, ***physical fields are not independent material substances***. And ***the field theory is not an independent mathematical formalism***. The field theory equations, which describe physical fields, in fact follow from the equations of mathematical physics describing material media.

The results of present paper, which disclose the foundations of unified and general field theories, have been obtained due to applying a mathematical apparatus of skew-symmetric differential forms defined on non-integrable manifolds (which existence was established by the author) and unique hidden properties of the mathematical physics equation (discovered by the author) that allow a description of evolutionary processes.

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Appendix 1

The equations of mathematical physics describing material media. In inertial frame of reference such equations are presented, for example, in monographs:

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