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ENERGETIC EFFECTS OF MAGNETIC FIELDS ON FREE ELECTRONS IN QUANTUM FIELD THEORY

CLAUDIO VERZEGNASSI

Politecnico di Ingegneria e Architettura
University of Udine
Udine
Italy
e-mail: claudio@ts.infn.it

AMeC (Association for Medicine and Complexity)
Trieste
Italy

Abstract

I consider the effects on the energy of free electrons produced by an applied magnetic field starting with the simplest case of a one electron state with a certain helicity. The general expression of the effects is fixed by the value of the helicity and by the values of the components of the associated magnetic potential. In the realistic case of a constant magnetic field, I show with a simple example that the energy shift can be expressed in terms of the components of the field in the space directions and strongly depends on the field orientation, which could enhance the effect or decrease it in a drastic way. The possibility of an extension of this result to a realistic organic system is considered as a future goal.

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1. Introduction

The effects of electromagnetic radiation on the human organism have been considered with great interest in the recent literature. In particular, the role of magnetic resonance has been deeply investigated (see, for instance, [1] and references therein).

The basic approach is based on the theoretical framework of non-relativistic quantum mechanics. In this scheme, the electromagnetic radiation communicates to the electrons a quantized amount of energy proportional to the radiation frequency. This generates quantum mechanics changes of the electron state, which are reflected in the organism and represents a powerful diagnostic tool.

A theoretical approach based on the relativistic quantum field theory has been proposed in [2] to describe coherent electron states of the water in the organism. This approach provided a description of a new coherent phase of the water tested in the laboratory.

In this paper, I investigate, in the framework of relativistic quantum field theory, the effects of an external magnetic field on the energy of an elementary electron state. This will be a continuation of the study of magnetic effects on the spin components of matter, in particular on a one-electron state, which has recently been performed in a quantum field theory framework [3]. I assume the conventional expression of the Hamiltonian H of a free electron field $\psi(x) = \psi(\vec{x}, t)$, which is required to satisfy the Dirac equation. This leads to computable changes of the four components, $\psi_1, \psi_2, \psi_3, \psi_4$, of the field under the action of an external magnetic field.

In particular, the effects of a classic electromagnetic potential $A_\mu = (A_0, \vec{A})$ on the field ψ can be described by using the minimal interaction prescription.

The final expression of the variation of the Hamiltonian, denoted ΔH_A, produced by the magnetic vector potential assumes then a precise form [4].

$$\Delta_{\vec{A}} H = 2|e|\left[\int d^3x\, \vec{A} \cdot \vec{J} - \frac{1}{m}\int d^3x \vec{H} \cdot \vec{\rho}_H\right], \tag{1}$$

where, in the Weyl (chiral) basis

$$\vec{J} = \bar{\psi}\vec{\gamma}\psi = \psi^\dagger\gamma^0\vec{\gamma}\psi = [2\,\mathrm{Re}(-\psi_1^*\psi_2 + \psi_3^*\psi_4),\ 2\,\mathrm{Im}(\psi_1^*\psi_2 + \psi_3^*\psi_4),$$
$$-\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3^*\psi_3 - \psi_4^*\psi_4], \tag{2}$$

and

$$\vec{H} = \vec{\Delta} \wedge \vec{A}, \quad \vec{\rho}_H = -\psi^\dagger\gamma^5\vec{\gamma}\psi = 2[\,\mathrm{Re}(\psi_1^*\psi_4 + \psi_2^*\psi_3),\ \mathrm{Im}(\psi_1^*\psi_4 - \psi_2^*\psi_3),$$
$$\mathrm{Re}(\psi_1^*\psi_3 - \psi_2^*\psi_4)]. \tag{3}$$

In the non-relativistic limit (NRL), $E \approx \vec{k}^2 / 2m$ and $\psi_1, \psi_2 >> \psi_3, \psi_4$, $\vec{\rho}_H \to 0$. This greatly simplifies Equation (1) which can now be written as:

$$\Delta_{\vec{A}} H_{NRL} = -4|e|\int d\vec{x}\vec{A} \cdot \vec{s}, \tag{4}$$

where $\vec{A}(\vec{x}, t)$ is the magnetic potential, e is the electron charge, and $\vec{s}$ is the spin density current, giving the total spin operator $\vec{S}$ of the field in the conventional expression:

$$\vec{S} = \int d\vec{x}\vec{s}. \tag{5}$$

In terms of the electron field $\psi(x, t)$, $\vec{s}$ has the form $(s_1 = s_{23}, s_2 = s_{31}, s_3 = s_{12})$:

$$s^{ab}(\vec{x}, t) = \frac{i}{2}\psi^\dagger(\vec{x}, t)\gamma^a\gamma^b\psi(\vec{x}, t), \tag{6}$$

where γ^a, γ^b are the usual gamma-matrices and a, b = 1, 2, 3. In the calculation that leads to Equation (4), the time t has been set equal to zero and the potential has been assumed to be reasonably weak, so that a first order perturbative expansion has been used for the obtained energy shift, and all these details are fully discussed in [4].

The aim of this paper is to compute the mean value of the operator (4) when the field system is that of a free one-electron state of momentum $\vec{k}_0$ and given spin. Following the conventional treatments, I shall identify the $\vec{k}_0$ direction with the z-axis and write:

$$\vec{k}_0 = (0,\ 0,\ k_0). \tag{7}$$

With $k_0 = |\vec{k}_0|$. Denoting as $\pm 1/2$ the possible eigenvalues of the electron spin along the magnetic field direction, I have considered, as it was done in a previous paper [3], a special electron state given by the combination:

$$|M_\pm >= \lambda_+ |\vec{k}_0^+ > + \lambda_- |\vec{k}_0^- >, \tag{8}$$

where $|\vec{k}_0^\pm >$ are the eigenstates of the spin component along the magnetic field with eigenvalue $\pm 1/2$ and λ_+, λ_+ are in my simple example real numbers such that

$$\lambda_+^2 + \lambda_-^2 = 1. \tag{9}$$

To perform a realistic but simple calculation, I have considered the case of a magnetic field $\vec{H}$ constant in the space and indicated it with the notation

$$\vec{H} = (H_1,\ H_2,\ H_3). \tag{10}$$

The magnetic potential $\vec{A}$ will then be given by the conventional formula

$$\vec{A} = \frac{1}{2}\left(\vec{H} \wedge \vec{r}\right), \quad \vec{r} = (x_1,\ x_2,\ x_3). \tag{11}$$

With this choice, Equation (4) becomes now

$$\Delta_{\vec{A}} H = -2|e| \int d\vec{x} \{ [H_2 x_3 - H_3 x_2] s_1 + [H_3 x_1 - H_1 x_3] s_2 + [H_1 x_2 - H_2 x_1] s_3 \}. \quad (12)$$

I want to compute the mean value of Equation (12) for an electron state defined by Equation (8). I shall call this quantity $< \Delta_{\vec{A}} H >_{\pm}$ and its value will be given by the ratio

$$< \Delta_{\vec{A}} H >_{\pm} = \frac{< M_{\pm} | \Delta_{\vec{A}} H | M_{\pm} >}{< M_{\pm} | M_{\pm} >}. \quad (13)$$

The calculation can be performed using the conventional Fourier transformed expression of $\psi(\vec{x})$ and $\psi^{\dagger}(\vec{x})$ involving the creation and destruction operators $a^{+}(\vec{k}_0)$, $a^{-}(\vec{k}_0)$ as we already did in [3]. This can be done from a mathematical point of view, but the involved *x*-integrals contain the position variables $x_{1,2,3}$ as Equation (12) shows. To perform a first reasonable evaluation, I have assumed that the relevant volume where the magnetic field interacts with the electron field is realistically limited, so that inside the *x*-integrals the space variables $x_{1,2,3}$ can be replaced by some suitable effective lengths named d_1, d_2, d_3 that are fixed by the specific case that one is treating. Accepting this effective assumption, the value of Equation (13) can be simply computed in a conventional way, and its expression becomes the following one:

$$\Delta_{\vec{A}} H = -|e| \{ [\lambda_{+}^{2} - \lambda_{-}^{2}] [H_1 d_2 - H_2 d_1] + 2\lambda_{+}\lambda_{-} [H_2 d_3 - H_3 d_2] \}. \quad (14)$$

Equation (14) is the main result of this paper. It provides already some relevant indications, valid for the most general type of magnetic field space orientation. The identification of the electron momentum with the *z*-axis can be abandoned, but for a first discussion I shall maintain it. I can then derive the following statements:

(I) For right-handed $(\lambda_+ = 1)$ or left-handed $(\lambda_- = 1)$ electrons, the magnetic effect is opposite, and only comes from the first term in Equation (14). This only contains the x_1 and x_2 components of the field. If the field direction is parallel to that of electron momentum, assumed in my treatment to be that of the x_3 axis, there is no magnetic effect on the electron energy. This is in fair agreement with the classic description of the Lorentz force on charged electric particles.

(II) I have defined as unpolarized state the one that has $\lambda_+ = \lambda_- = 1/\sqrt{2}$. In this case, the energy shift is produced by the second term of Equation (14), where only the field components along x_2 and x_3 remain. If the field direction is that of the x_1 axis, there is no effect on the unpolarized state in my treatment.

I want to conclude this paper providing a possibly relevant discussion which starts from the previous statement. With this aim, I consider the case of an initial magnetic field that only has a x_1 component and define it as

$$\vec{H}^{(in)} = \left(H_1^{(in)}, 0, 0\right). \tag{15}$$

This field will produce an energy shift on right and left handed electrons, given by the expression

$$\Delta_{\vec{A}} H = -|e|[\lambda_+^2 - \lambda_-^2] H_1^{(in)} d_2. \tag{16}$$

If one applies this field to the unpolarized state there will be no effect. To obtain an effect, it will be necessary and sufficient to change the direction of the initial field. For instance, one can produce a new field $\vec{H}^{(new)}$ performing a rotation of the original $\vec{H}^{(in)}$ in the (x_1, x_2) plane with a rotation angle θ_{12}. This will produce a field

$$\vec{H}^{(new)} = \left(H_1^{(new)}, H_2^{(new)}, 0\right), \tag{17}$$

where

$$H_1^{(new)} = H_1^{(in)} \sin \theta_{12}, \tag{18}$$

$$H_2^{(new)} = H_2^{(in)} \cos \theta_{12}. \tag{19}$$

The effect on the unpolarized state will now be:

$$< \Delta_{\vec{A}} H >_{(unpol)}^{(new)} = - 2|e|\lambda_+ \lambda_- H_1^{(in)} d_3 \cos \theta_{12}. \tag{20}$$

One sees that the nonzero effect, which has been obtained performing a rotation of the previous field (that provided a vanishing effect) has a size whose ratio with the original right or left handed effect produced by the initial field is

$$\frac{| < \Delta_{\vec{A}} H >_{\pm}^{(in)} |}{< \Delta_{\vec{A}} H >_{(unpol)}^{(new)}} = \frac{d_2}{d_3 \cos \theta_{12}}. \tag{21}$$

Equation (21) shows that the direction of the magnetic field is indeed relevant: in my simple example, one can generate an effect starting from a field that produced no effects and rotating it in a proper way. It seems to me that this prediction is a specific result of my quantum field theory approach.

2. Conclusion

The previous results have been obtained for a very special case. The obvious next step would be to investigate whether similar features might appear in a more relevant case, possibly and particularly related to the effects on a human organic system. This is certainly beyond the purposes of my paper. Still, I believe that in a "*fractal*" view [5], the possibility that special features of an elementary component are reproduced by the relevant overall system cannot be excluded. Work in this direction, with a set of interested colleagues (P. Kurian and G. Vitiello) is in progress.

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