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NEW INTERVAL VALUE INTUITIONISTIC FUZZY SETS

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Abstract

In this paper, a generalized interval value intuitionistic fuzzy set (GIVIFS_B) is proposed. It is showed that Atanassov's interval value intuitionistic fuzzy set is a special case of this new one. Further, some important notions and basic algebraic properties of GIVIFS_B are discussed.

1. Introduction

Zadeh's fuzzy sets [33] characterized only by membership function, where the values membership function are numbers belong to [0, 1]. Zadeh [32] and Turksen [26] introduced the concept of interval valued fuzzy subsets (IVFS), where the values of the membership functions are intervals of numbers instead of the numbers. Later on, Atanassov [5] introduced some operations over interval-valued fuzzy set.

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Atanassov [3] introduced the concept of intuitionistic fuzzy sets (IFSs), which is a generalization of fuzzy subsets and defines new operations on IFSs. In this extension, IFSs are defined using degree of membership belong to [0, 1] and a degree of non-membership belong to [0, 1], under the constraint that the sum of two degrees does not exceeds one. Following the definition IFS, Atanassov and Gargov [4] introduced interval valued intuitionistic fuzzy sets (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFSs is that the values of its membership function and non membership function are intervals at [0, 1] rather than exact numbers. The IVIFSs has been studied and applied in a variety of fields as pattern recognition, medical diagnosis, decision making, data mining, conflict analysis algebra and so on.

Xu [27, 28] developed some arithmetic aggregation operators and some geometric aggregation operators of IVIFS, to be used in decision making. Ahn et al. [2] IVIFS theory has been applied to make a diagnosis of headache as a new approach on decision support practice in medicine. Ye [30] and Jing and Min [20] studied the entropy of the interval-valued intuitionistic fuzzy sets and its applications. Hu and Li [18] investigate the relationship between entropy and similarity of interval valued intuitionistic fuzzy sets.

Bustince and Burillo [12], Hong [19] and Zeng and Wang [34] introduced the concepts of correlation and correlation coefficient of interval-valued intuitionistic fuzzy sets. Park et al. [24] extend three methods for measuring distances between IVFSs to IVIFSs. Xu [29] introduced some relations and operations of interval-valued intuitionistic fuzzy numbers and define some types of interval-valued intuitionistic fuzzy matrices for group decision making. Bhowmik and Pal [10, 11] defined generalized interval-valued intuitionistic fuzzy set (GIVIFS) and presented various properties of it. Zhenhua et al. [35] introduced generalized interval-valued intuitionistic fuzzy sets with parameters. Chen et al. [13], Yue [31] and Li [14, 15, 16] presented methods for Multicriteria fuzzy decision making based on IVIFS. Mondal and Samanta [23] studied the topological properties and the category of topological spaces of IVIFSs. Adak and Bhowmik [1] defined interval cut-set of interval-valued intuitionistic fuzzy sets. Mishra and Pal [22] product of interval valued intuitionistic fuzzy graph. Baloui and Nadaraja [9] introduced a generalization of the IFS. In this paper, the main objective is introduce a new generalized of interval value intuitionistic fuzzy sets and also define some operations and their properties.

2. Preliminaries

In this section, we give some definition. Let X be a non-empty set.

Definition 2.1 (Atanassov [3])**.** An IFS A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership and non-membership functions of A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 (Atanassov and Gargov [4])**.** Interval value intuitionistic fuzzy sets (IVIFS) A in X, is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle | x \in X\}$, where the functions $M_A(x) : X \to [I]$ and $N_A(x) : X \to [I]$, denote the degree of membership and degree of non membership of A, respectively, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$, $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, $0 \leq M_{AU}(x) + N_{AU}(x) \leq 1$ for each $x \in X$.

Definition 2.3 (Baloui Jamkhaneh and Nadarajah [9])**.** Generalized intuitionistic fuzzy sets (GIFS_B) A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$, denote the degree of membership and degree of non membership functions of A, respectively, and $0 \leq \mu_A(x)^\delta + \upsilon_A(x)^\delta \leq 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}$, $n = 1, 2, \ldots, N$.

Definition 2.4. Let $[I]$ be the set of all closed subintervals of the interval $[0,1]$ and $M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)] \in [I]$, then $N_A(x) \leq M_A(x)$ if and only if $N_{AL}(x) \leq M_{AL}(x)$ and $N_{AU}(x) \leq M_{AU}(x)$.

Definition 2.5. Let $[I]$ be the set of all closed subintervals of the interval $[0, 1]$ and $M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$, $f : [0, 1] \to [0, 1]$, then $f(M_A(x)) = [f(M_{AL}(x)), f(M_{AU}(x))]$.

3. Generalized Interval Value Intuitionistic Fuzzy Sets

Definition 3.1. Let X be a non empty set. Generalized interval value intuitionistic fuzzy sets $(\mathrm{GIVIFS_B})$ A in X, is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle | x \in X\}$, where the functions $M_A(x) : X \to [I]$ and $N_A(x) : X \to [I]$, denote the degree of membership and degree of non membership of A, respectively, and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$, $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, where $0 \leq M_{AU}(x)^{\delta} + N_{AU}(x)^{\delta} \leq 1$, for each $x \in X$ and $\delta = n$ or $\frac{1}{n}$, $n = 1, 2, \ldots, N$. The collection of all $GIVIFS_B(\delta)$ is denoted by $GIVIFS_B(\delta, X)$.

Remark 3.1. It is obvious that for all real numbers $\alpha, \beta \in [0, 1]$.

(i) If $0 \leq \alpha + \beta \leq 1$ and $\delta \geq 1$, then we have $0 \leq \alpha^{\delta} + \beta^{\delta} \leq 1$, with this consideration if $A \in IVIFS$, then $A \in GIVIFS_B$.

(ii) If $0 \leq \alpha^{\delta} + \beta^{\delta} \leq 1$ and $\delta \leq 1$, then $0 \leq \alpha + \beta \leq 1$, with this consideration if $A \in GIVIFS_B$, then $A \in IVIFS$.

(iii) If $\delta_1 \leq \delta_2$, then $\alpha^{\delta_2} \leq \alpha^{\delta_1}$ and $\beta^{\delta_2} \leq \beta^{\delta_1}$. It follows that $GIVIFS_B(\delta_1) \subset GIVIFS_B(\delta_2)$.

Definition 3.2. Let X be a non-empty set. Let A and B be two $GIVIFS_Bs$ such that

$$A = \{\langle x,\ M_A(x),\ N_A(x)\rangle : x \in X\},$$

$$B = \{\langle x,\ M_B(x),\ N_B(x)\rangle : x \in X\},$$

$$M_A(x) = [M_{AL}(x),\ M_{AU}(x)], \quad N_A(x) = [N_{AL}(x),\ N_{AU}(x)],$$

$$M_B(x) = [M_{BL}(x),\ M_{BU}(x)], \quad N_B(x) = [N_{BL}(x),\ N_{BU}(x)],$$

define the following relations and operations on A and B:

(i) $A \subset B$ if and only if $M_A(x) \le M_B(x)$ and $N_A(x) \ge N_B(x)$, $\forall x \in X$,

(ii) $A = B$ if and only if $M_A(x) = M_B(x)$ and $N_A(x) = N_B(x)$, $\forall x \in X$,

(iii) $A \cup B = \{\langle x, [\max(M_{AL}(x),\ M_{BL}(x)),\ \max(M_{AU}(x),\ M_{BU}(x))],$

$[\min(N_{AL}(x),\ N_{BL}(x)),\ \min(N_{AU}(x),\ N_{BU}(x))]\rangle : x \in X\},$

(iv) $A \cap B = \{\langle x, [\min(M_{AL}(x),\ M_{BL}(x)),\ \min(M_{AU}(x),\ M_{BU}(x))],$

$[\max(N_{AL}(x),\ N_{BL}(x)),\ \max(N_{AU}(x),\ N_{BU}(x))]\rangle : x \in X\},$

(v) $A + B = \{\langle x, [M_{AL}(x)^{\delta} + M_{BL}(x)^{\delta} - M_{AL}(x)^{\delta} M_{BL}(x)^{\delta},\ M_{AU}(x)^{\delta}$

$+ M_{BU}(x)^{\delta} - M_{AU}(x)^{\delta} M_{BU}(x)^{\delta}],\ [N_{AL}(x)^{\delta}.N_{BL}(x)^{\delta},$

$N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta}]\rangle : x \in X\},$

therefore,

$$2A = \left\{\langle x, \left[1 - (1 - M_{AL}(x)^{\delta})^2, 1 - (1 - M_{AU}(x)^{\delta})^2\right], [N_{AL}(x)^{2\delta}.N_{AU}(x)^{2\delta}]\rangle : x \in X\right\},$$

and

$$nA = \left\{\langle x, \left[1 - (1 - M_{AL}(x)^{\delta})^n, 1 - (1 - M_{AU}(x)^{\delta})^n\right], [N_{AL}(x)^{n\delta}.N_{AU}(x)^{n\delta}]\rangle : x \in X\right\},$$

(vi)

$$A.B = \{\langle x, [M_{AL}(x)^{\delta}.M_{BL}(x)^{\delta}, M_{AU}(x)^{\delta}.M_{BU}(x)^{\delta}], [N_{AL}(x)^{\delta} + N_{BL}(x)^{\delta}$$

$$- N_{AL}(x)^{\delta} N_{BL}(x)^{\delta}, N_{AU}(x)^{\delta} + N_{BU}(x)^{\delta} - N_{AU}(x)^{\delta} N_{BU}(x)^{\delta}]\rangle : x \in X\},$$

therefore,

$$A^2 =$$

$$\left\{\langle x, [M_{AL}(x)^{2\delta}, M_{AU}(x)^{2\delta}], [1-(1-N_{AL}(x)^{\delta})^2, 1-(1-N_{AU}(x)^{\delta})^2]\rangle : x \in X\right\},$$

and

$$A^n =$$

$$\left\{\langle x, [M_{AL}(x)^{n\delta}, M_{AU}(x)^{n\delta}], [1-(1-N_{AL}(x)^{\delta})^n, 1-(1-N_{AU}(x)^{\delta})^n]\rangle : x \in X\right\},$$

(vii) $\overline{A} = \{\langle x, N_A(x), M_A(x)\rangle : x \in X\}$.

Proposition 3.1. *Let* $A, B, C \in GIVIFS_B$, *we have*

(i) $\overline{\overline{A}} = A$,

(ii) $\overline{A \cup B} = \overline{A} \cap \overline{B}$,

(iii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$,

(iv) $A \subset B, B \subset C \Rightarrow A \subset C$.

Proof. Proof is obvious.

Proposition 3.2. *Let* $A, B \in GIVIFS_B$, *we have*

(i) $A \cup B \in GIVIFS_B$,

(ii) $A \cap B \in GIVIFS_B$,

(iii) $\delta \geq 1 \Rightarrow A + B \in GIVIFS_B$,

$\delta < 1 \Rightarrow A + B \in IVIFS$,

(iv) $\delta \geq 1 \Rightarrow A.B \in GIVIFS_B$,

$\delta < 1 \Rightarrow A.B \in IVIFS$,

Proof. (i) Let $\max(M_{AU}(x), M_{BU}(x)) = M_{AU}(x)$, since $\min(N_{AU}(x), N_{BU}(x)) \leq N_{AU}(x)$, we have

$$0 \leq M_{(A\cup B)U}(x)^{\delta} + N_{(A\cup B)U}(x)^{\delta},$$

$$= (\max(M_{AU}(x), M_{BU}(x)))^{\delta} + (\min(N_{AU}(x), N_{BU}(x)))^{\delta},$$

$$= M_{AU}(x)^{\delta} + (\min(N_{AU}(x), N_{BU}(x)))^{\delta},$$

$$\leq M_{AU}(x)^{\delta} + N_{AU}(x)^{\delta} \leq 1.$$

If $\max(M_{AU}(x), M_{BU}(x)) = M_{BU}(x)$, since $\min(N_{AU}(x), N_{BU}(x)) \leq N_{BU}(x)$, we have

$$0 \leq M_{(A\cup B)U}(x)^{\delta} + N_{(A\cup B)U}(x)^{\delta}$$

$$= (\max(M_{AU}(x), M_{BU}(x)))^{\delta} + (\min(N_{AU}(x), N_{BU}(x)))^{\delta}$$

$$= M_{BU}(x)^{\delta} + (\min(N_{AU}(x), N_{BU}(x)))^{\delta}$$

$$\leq M_{BU}(x)^{\delta} + N_{BU}(x)^{\delta} \leq 1.$$

The proof is completed. Proof of (ii) is similar (i).

(iii) We know

$$A + B = \{\langle x, [M_{AL}(x)^{\delta} + M_{BL}(x)^{\delta} - M_{AL}(x)^{\delta} M_{BL}(x)^{\delta}, M_{AU}(x)^{\delta}$$

$$+ M_{BU}(x)^{\delta} - M_{AU}(x)^{\delta} M_{BU}(x)^{\delta}], [N_{AL}(x)^{\delta} \cdot N_{BL}(x)^{\delta},$$

$$N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta}]\rangle : x \in X\},$$

then

$$M_{(A+B)U}(x)^{\delta} + N_{(A+B)U}(x)^{\delta}$$

$$= (M_{AU}(x)^{\delta} + M_{BU}(x)^{\delta} - M_{AU}(x)^{\delta} M_{BU}(x)^{\delta})^{\delta} + (N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta})^{\delta}$$

$$= (M_{AU}(x)^{\delta}(1 - M_{BU}(x)^{\delta}) + M_{BU}(x)^{\delta})^{\delta} + (N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta})^{\delta} \geq 0,$$

and

$$M_{(A+B)U}(x)^{\delta} + N_{(A+B)U}(x)^{\delta}$$

$$= (M_{AU}(x)^{\delta} + M_{BU}(x)^{\delta} - M_{AU}(x)^{\delta} M_{BU}(x)^{\delta})^{\delta} + (N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta})^{\delta}$$

$$\leq (\left(1 - N_{AU}(x)^{\delta}\right) + (1 - N_{BU}(x)^{\delta}) - \left(1 - N_{AU}(x)^{\delta}\right)(1 - N_{BU}(x)^{\delta})^{\delta}$$

$$+ (N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta})^{\delta}$$

$$= \left(1 - N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta}\right)^{\delta} + (N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta})^{\delta}$$

$$= (1 - u)^{\delta} + u^{\delta}, \quad u = N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta}.$$

If $\delta \geq 1$, then $(1 - u)^{\delta} + u^{\delta} \leq 1$, hence $A + B \in IVIFS_B$. If $\delta < 1$, then $(1 - u)^{\delta} + u^{\delta} \leq 1$, if and only if $N_{AU}(x) = 0$ or $N_{BU}(x) = 0$. But for any δ, we have

$$M_{(A+B)U}(x) + N_{(A+B)U}(x)$$

$$= (M_{AU}(x)^{\delta} + M_{BU}(x)^{\delta} - M_{AU}(x)^{\delta} M_{BU}(x)^{\delta}) + (N_{AU}(x)^{\delta}.N_{BU}(x)^{\delta})$$

$$\leq (M_{AU}(x)^{\delta} + M_{BU}(x)^{\delta} - M_{AU}(x)^{\delta} M_{BU}(x)^{\delta})$$

$$+ (1 - M_{AU}(x)^{\delta}).(1 - M_{BU}(x)^{\delta}) = 1,$$

hence $A + B \in IVIFS$.

Proof of (iv) is similar (iii).

Proposition 3.3. *For every* $GIVIFS_B$ A, *we have*

(i) $m \geq n \to A^m \subset A^n$,

(ii) $m \geq n \to nA \subset mA$,

(iii) $A^n = \overline{\overline{nA}}$,

where m and n are both positive number.

Proof. (i)

$A^n =$

$\{\langle x, [M_{AL}(x)^{n\delta}, M_{AU}(x)^{n\delta}], [1-(1-N_{AL}(x)^{\delta})^n, 1-(1-N_{AU}(x)^{\delta})^n]\rangle : x \in X\}$,

$A^m =$

$\{\langle x, [M_{AL}(x)^{m\delta}, M_{AU}(x)^{m\delta}], [1-(1-N_{AL}(x)^{\delta})^m, 1-(1-N_{AU}(x)^{\delta})^m]\rangle : x \in X\}$.

Since $m \geq n$, then $M_{AU}(x)^{n\delta} \geq M_{AU}(x)^{m\delta}$, $M_{AL}(x)^{n\delta} \geq M_{AL}(x)^{m\delta}$ hence $M_{A^n}(x) \geq M_{A^m}(x)$. Also since $N_{AL}(x) \leq 1$, $N_{AU}(x) \leq 1$, then $(1-N_{AU}(x)^{\delta})^m \leq (1-N_{AU}(x)^{\delta})^n$, $(1-N_{AL}(x)^{\delta})^m \leq (1-N_{AL}(x)^{\delta})^n$ hence $1-(1-N_{AU}(x)^{\delta})^n \leq 1-(1-N_{AU}(x)^{\delta})^m$, $1-(1-N_{AL}(x)^{\delta})^n \leq 1-(1-N_{AL}$ $(x)^{\delta})^m$, hence $N_{A^n}(x) \leq N_{A^m}(x)$.

Proof is complete. Proof of (ii) is similar (i). Proof of (iii) is clearly.

Proposition 3.4. *Let* $A, B \in GIVIFS_B$, *we have*

(i) $A \subset B \to nA \subset nB$,

(ii) $A \subset B \to A^n \subset B^n$,

(iii) $(A \cup B)^n = A^n \cup B^n$,

(iv) $(A \cap B)^n = A^n \cap B^n$,

(v) $n(A \cup B) = nA \cup nB$,

(vi) $n(A \cap B) = nA \cap nB$.

Proof. (i) Since $A \subset B$, then $M_A(x) \le M_B(x)$ hence

$$M_A(x)^{\delta} \le M_B(x)^{\delta} \Rightarrow 1 - M_B(x)^{\delta} \le 1 - M_A(x)^{\delta} \Rightarrow (1 - M_B(x)^{\delta})^n \le (1 - M_A(x)^{\delta})^n,$$

finally,

$$1 - (1 - M_A(x)^{\delta})^n \le 1 - (1 - M_B(x)^{\delta})^n \Rightarrow M_{nA}(x) \le M_{nB}(x).$$

Also since $A \subset B$, then $N_B(x) \le N_A(x)$ hence

$$N_B(x)^{n\delta} \le N_A(x)^{n\delta} \Rightarrow N_{nB}(x) \le N_{nA}(x),$$

proof is complete.

(ii)

$$A \subset B \Rightarrow \overline{B} \subset \overline{A} \Rightarrow n\overline{B} \subset n\overline{A} \Rightarrow \overline{n\overline{A}} \subset \overline{n\overline{B}} \Rightarrow A^n \subset B^n.$$

(iii)

$$(A \cup B)^n = \{\langle x, \left[(\max(M_{AL}(x), M_{BL}(x))^{n\delta}, (\max(M_{AU}(x), M_{BU}(x))^{n\delta}\right], [1 - (1 - \min(N_{AL}(x), N_{BL}(x))^{\delta})^n, 1 - \left(1 - \min(N_{AU}(x), N_{BU}(x))^{\delta}\right)^n \Big]\rangle : x \in X\}$$

$$= \{\langle x, [\max(M_{AL}(x)^{n\delta}, M_{BL}(x)^{n\delta}), \max(M_{AU}(x)^{n\delta}, M_{BU}(x)^{n\delta})], [1 - (1 - \min(N_{AL}(x)^{\delta}, N_{BL}(x)^{\delta}))^n, 1 - (1 - \min(N_{AU}(x)^{\delta}, N_{BU}(x)^{\delta}))^n]\rangle : x \in X\}$$

$$= \{\langle x, [\max(M_{AL}(x)^{n\delta}, M_{BL}(x)^{n\delta}), \max(M_{AU}(x)^{n\delta}, M_{BU}(x)^{n\delta})], [1 - (\max(1 - N_{AL}(x)^{\delta}, 1 - N_{BL}(x)^{\delta})^n, 1 - \left(\max(1 - N_{AU}(x)^{\delta}, 1 - N_{BU}(x)^{\delta})\right)^n \Big]\rangle : x \in X\}\}$$

$$=\{\langle x,[\max(M_{AL}(x)^{n\delta},M_{BL}(x)^{n\delta}),\max(M_{AU}(x)^{n\delta},M_{BU}(x)^{n\delta})],$$

$$[\min(1-(1-N_{AL}(x)^{\delta})^{n},1-(1-N_{BL}(x)^{\delta})^{n}),$$

$$\min(1-(1-N_{AU}(x)^{\delta})^{n},1-(1-N_{BU}(x)^{\delta})^{n})]\rangle : x\in X\}\}$$

$$= A^n \cup B^n.$$

Proof (iv) is similar (iii).

(v)

$$n(A\cup B)$$

$$=\{\langle x,\left[1-(1-\max(M_{AL}(x),M_{BL}(x))^{\delta})^{n},(1-(1-\max(M_{AU}(x),M_{BU}(x))^{\delta})^{n}\right],$$

$$[\min(N_{AL}(x),\ N_{BL}(x))^{n\delta}\ \min(N_{AU}(x),\ N_{BU}(x))^{n\delta}]\rangle : x\in X\}$$

$$=\{\langle x,[1-(1-\max(M_{AL}(x)^{\delta},M_{BL}(x)^{\delta}))^{n},$$

$$(1-(1-\max\left(M_{AU}(x)^{\delta},M_{BU}(x)^{\delta}\right))^{n}\Big],$$

$$[\min(N_{AL}(x),\ N_{BL}(x))^{n\delta},\ \min(N_{AU}(x),\ N_{BU}(x))^{n\delta}]\rangle : x\in X\}$$

$$=\{\langle x,\Big[1-\min((1-M_{AL}(x)^{\delta})^{n},(1-M_{BL}(x)^{\delta})^{n}),$$

$$1-\min((1-M_{AU}(x)^{\delta})^{n},(1-M_{BU}(x)^{\delta})^{n}\Big]$$

$$[\min(N_{AL}(x),\ N_{BL}(x))^{n\delta},\ \min(N_{AU}(x),\ N_{BU}(x))^{n\delta}]\rangle : x\in X\}$$

$$=\{\langle x,[\max(1-(1-M_{AL}(x)^{\delta})^{n},1-(1-M_{BL}(x)^{\delta})^{n}),$$

$$\max(1-(1-M_{AU}(x)^{\delta})^{n},1-(1-M_{BU}(x)^{\delta})^{n}],[\min(N_{AL}(x),$$

$$N_{BL}(x))^{n\delta},\min(N_{AU}(x),\ N_{BU}(x))^{n\delta}]\rangle : x\in X\}$$

$$= nA\cup nB.$$

Proof (vi) is similar (v).

5. Conclusion

We have introduced Baloui's generalized interval value intuitionistic fuzzy set $(\mathrm{GIVIFS_B})$ as an extension to the interval value intuitionistic fuzzy set. The basic algebraic properties on $\mathrm{GIVIFS_B}$ are also presented. Some operations on $\mathrm{GIVIFS_B}$ are defined and their relationship are proved. $\mathrm{GIVIFS_B}$ is more comprehensive and practical than IVIFS in coping with fuzziness and uncertainty. A list of open problems is as follows: definition of generalized interval value intuitionistic fuzzy number and norms, distances, metrics, metric space, similarity measures, new operators and etc over $\mathrm{GIVIFS_B}$ and study their properties.

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