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SIMPLE INCONSISTENCY OF SIMPLE REALIST QM INTERPRETATIONS AND RELATED MATHEMATICAL THEORIES

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Abstract

A few realist theories in physics and mathematics are analyzed from a constructivist viewpoint. Technical notions like fixed-point theorems and homeomorphisms are employed to pinpoint the logical and mathematical import of the critique.

1. Introduction: Realism in Physics

G. 't Hooft defends a realist interpretation of QM in his long working paper "*The Cellular Automaton Interpretation of Quantum Mechanics*" [9]. The author advocates a deterministic or superdeterministic view of the universal wave function for the Schrödinger equation that would not ramify as in Everett's many-universes interpretation -- they are

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practically uncountable, as ‘t Hooft says in his informal idiom --, but would give rise to a unique universe, a fluctuating vacuum filled with « solid » quanta or « fluid » particles that would obey a law-like determinism evolving from a given fundamental field, such as a scalar field or a quantum (true or false) vacuum. The entire universe of the wave function has a real deterministic *ontological* basis in Hilbert space following ‘t Hooft. But ‘t Hooft does not say that such a basis must have a finite cardinality or at most an infinite countable cardinality $\aleph_0$. The universal wave function ψ with its values in **R** or **C** has however the uncountable cardinality $2^{\aleph_0} = \beth_1$ (beth number), the power of the continuum -- the continuum hypothesis $(\mathrm{CH})\, c = 2^{\aleph_0} = \aleph_1$ is not needed in our discussion --. The infinite-dimensional Hilbert spaces that are used in the separable space topology are all homeomorphic in countable cardinality (of a countable orthonormal basis): a Hilbert space of uncountable cardinality $2^{\aleph_0}$ is not separable and the notion of homeomorphism here has the usual definition of a bicontinuous bijection, that is, if the function $f(x)$ is continuous, its inverse $f^{-1}(x)$ is also continuous. The same holds in the more general Fock spaces for the second quantization in quantum field theory with its many-particle systems and von Neumann C^* separable algebras, while larger spaces like infinite-dimensional Banach spaces equipped with an uncountable Hamel basis provided by the axiom of choice smash all dimensions in one *indistinct* all-encompassing continuum. Separable Hausdorff spaces as functional spaces have a cardinality higher than the continuum ***c***, that is, $\beth_2$ or 2^c, but their Hausdorff dimension d for regular metric spaces like Euclidean spaces $\mathbf{R}^n$ or $\mathbf{R}^\omega$ (the ordinal of $\aleph_0$) corresponds to the finite or countable dimensions of Hilbert spaces -- the Hausdorff dimension d for irregular finite or countable metric spaces is 0 --. Hilbert spaces of finite or $\aleph_0$ dimensions are also Banach spaces, but they are the natural setting for self-adjoint operators and observables designed to capture the finite probability values of the Born rule $(\psi^*\psi = |\psi|^2)$ for *actual* concrete measurements. There is no bijection between $\aleph_0$ and $2^{\aleph_0}$, the set of all finite and infinite subsets or combinations of $\aleph_0$ quantum states. So the universal wave function is inaccessible or unassailable from the $\aleph_0$ infinite-dimensional Hilbert space perspective. The case of finite-dimensional Hilbert space does not fare better and is still worse for ‘t Hooft cellular automata or finite lattice-theoretic devices, since Hilbert spaces are not homeomorphic over different dimensions $m < n$ and cannot reach a unique infinite-$\aleph_0$ dimensional Hilbert space. What this means is the mathematical fact from transfinite arithmetic that if the universal wave function ψ could be *realized*, the one universe would be in all its states at once or *in toto simul sub specie aeternitatis* in contradiction to ‘t Hooft view that there is only one state of the universal wave function at any instant. In both cases though, such a universe would be indeed immeasurable – with no experiment whatsoever to measure anything and there would be no need at all for a measurement theory, observers, no Hilbert space of observables, not a quantum bit of quantum logic and no-go theorems, as ‘t Hooft notes, and for that matter ultimately no QM and no physics at all! But there could be a metaphysical spin-off with the Dutch philosopher Baruch Spinoza. One may repeat this little exercise in inconsistency in its Spinozistic version in Pars 1 of the *Ethica*: the infinite attributes of the infinite substance (*Deus sive Natura*), extension (*res extensa*) and thought (*res cogitans*) must be not only isomorphic, but homeomorphic according to Proposition 7 of the *Ethica*:

Ordo et connexio idearum est ac ordo et connexio rerum.

So, if one supposes a deterministic universe compatible with an infinite Hilbert space of quantum states, Nature as *res extensa* (*Natura*) or God as *res cogitans* (*Deus* or God’s mind) with isomorphic $\aleph_0$

cardinalities cannot comprehend the $2^{\aleph_0}$ cardinality of the wave function continuum. In a Spinozistic perspective, the holographic principle promoted by 't Hooft and others would be a holomorphic (continuous) *distorting mirror*, but not a homeomorphic (bicontinuous) image in the passage of a 3-dimensional universe (made of 1-dimensional strings and higher-dimensional branes?) to a conformally mapped 2-dimensional boundary. In the end, even God's infinite mind or other infinite minds do not have access to homeomorphic perfect knowledge or omniscience though omnipresent with respect to different finite or infinite (countable or uncountable or even indeterminate) dimensions…. Still, radical simple realism could claim access to the wave function in a dimensionless universe without four-dimensional space-time and higher dimensions, but that would imply an extra scientific, poetic or mystical experience, for an other philosopher Ludwig Wittgenstein perhaps in the sense of Proposition 6.45 of his *Tractatus logico-philosophicus* « The experience (*Anschauung*) of the world *sub specie aeterni* is the experience of the world as a closed whole (*begrenztes Ganzes*). The feeling (*Gefühl*) of the world as a closed whole is the mystical ». Such a closed whole however becomes a closed *n*-dimensional space in physical cosmology.

A superdeterministic universe is equally inaccessible in Everett's and in 't Hooft's versions of QM as well as in Spinoza's metaphysics. For the philosopher-mathematician Leibniz in his *Theodicy* (par. 29), possible worlds as combinations of an « infinity of infinities » in God's mind, as he says, could amount to $2^{\aleph_0}$ in Cantorian transfinite arithmetic, but it is only God who has the combinatory power of choosing the best possible world among all those combinations. On the side of philosophical logic, the philosophical theology of possible worlds semantics in modal logic (Lewis or Kripke) with a *designated* world – the *actual* world possibly as the *best* combination -- is hardly a substitute here.

This little exercise has been performed with Cantor's diagonal argument for his power set theorem in transfinite set theory. The present author as a constructivist logician is not an endorser of the Cantorian stance! The authors under discussion adopt the Cantorian distinction between countable and uncountable infinities. Of course, Spinoza who has inspired Cantor, cannot be counted as a protagonist here, but his idea of a homeomorphism or perfect accord between the infinities of extension and thought puts him on the $\aleph_0$ countable cardinality line. It is worth noticing that it is another Dutchman, the mathematician Brouwer, who has shown against Cantor that if all continua are isomorphic in any dimension, that is of the cardinality of the continuum *c*, they are not homeomorphic (in bicontinuous bijection) in different finite or infinite dimensions. Brouwer would reject also the diagonal argument and would not grant the continuum a definite cardinality since it is a process in becoming « *ein Prozess im Werden* » of an *indeterminate* mathematical substratum that could be determined only by choice sequences of a creative subject, as he conceived it. In the same line of thought, Hermann Weyl had insisted on a constructive notion of the mathematical continuum and he even introduced in the 1920's the idea of a physical continuum of infinitely novel becoming fueled by decisions « *Entscheidungen* » in a probabilistic universe. Weyl would need only the combinations allowed by the binomial distribution for the discrete probability distribution

$$p(k) = \binom{n}{k} p^k q^{n-k},$$

with the expansion of the binomial coefficient

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n \text{ for } \frac{n!}{k!\,(n-k)!} = {}_nC_k,$$

for the combinations C and the power set 2^n of a set of n finite elements or experiments. Finite experiments include actual experiments and gedanken experiments that are free choices of the experimenters as accounted for in the recent Conway-Kochen theory (see [5] and [6]).

Bell's theorem on local hidden-variable theories and its accompanying test experiments belong to this category and their loopholes – among them the locality or communication loophole and the so-called free-choice loophole -- seem to have been put to rest in recent experiments (by Hanson among others) to the point that Zeilinger has claimed the final dismissal of local realism in favour of quantum entanglement which is, in my view, a purely combinatorial particle theory. In that context, superdeterminism looks like a universal local realism where free choice is blinded by an entropic loss of information. In the Weylian worldview as in the Conway-Kochen theory, free choice in an essential ingredient of a chaotic, stochastic universe and the local observer disposes of all the necessarily finite information available. This means that the total information contained in the universal wave function is in practice forever unavailable from the experimenter's side. From the theoretician's side, we have seen that such information is beyond physics and mathematics and there is no possible trade-off with metaphysics where no physics, experimental or theoretical, is admitted, nor is there any place for constructive mathematics in that Platonic heaven. Superdeterminism has been shown here to be inconsistent on both counts, transfinite arithmetic and constructive mathematics.

2. Realism in Mathematics

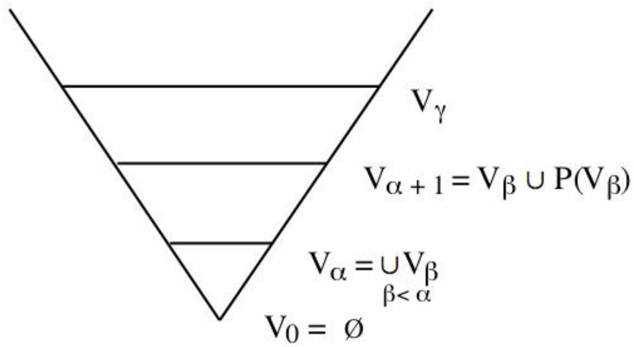
To come back now to a more mundane way of speaking in a mathematical idiom, let us see if we could use the homeomorphism idea (or ideal) to evaluate the contents of Cantor's paradise or Grothendieck's U-topia. I recall that U-topia is the grand assemblage of Grothendieck's U-topoi (with Tarski- Grothendieck universes) while Cantor's paradise is the habitat of the transfinite arithmetic of ordinals and cardinals described by the omegas ω and the alephs $\aleph$. The omegas as limit ordinals run like this:

$$\omega (\text{or } \omega_0), \omega_1, \omega_2, \ldots,$$

the alephs follow suit:

$$\aleph_0, \aleph_1, \aleph_2, \dots, \aleph_\omega.$$

Higher set theory, higher category theory with infinity or ∞-categories and higher topoi theory seem to require a (strongly) inaccessible cardinal in the cumulative ordinal hierarchy of Zermelo-Fraenkel with Choice (ZFC) set theory on level V_γ which looks like this:



where γ is the inaccessible cardinal, V_0 the null set, $\cup$ union, P stands for the power set and the α, β, γ for ordinals – here a cardinal is the least ordinal equinumerous to the well-ordered set of all smaller ordinals as defined by von Neumann--. Let us consider those ordinal stages as dimensions in the set-theoretic hierarchical universe (or multiverse) based on the axiom of foundation which essentially says that there is no infinitely descending sequence of ordinals from V_γ for the membership relation $\in$ (V_γ has a transitive model). Inaccessible means not accessible from previous stages by the union and power set operations: needless to say, the existence of inaccessible cardinals and other higher (larger) cardinals is unprovable in ZFC, for example, the continuum with $c = \aleph_1 = \beth_1$ has the ordinal ω_1 which clearly has inaccessible status.

Voevodsky in his univalent homotopy theory with the axiom «homotopic equivalence = identity», Mochizuki in his virtual inter-universal geometry and Martin-Löf in his transfinite constructive type theory all claim to reach for an inaccessible cardinal. The reflection principle in ZF set theory says that the properties of the universe V of all sets or the superuniverse Ω of all universes can be reflected in an inaccessible cardinal γ of the cumulative ordinal hierarchy. Since homotopy is limited to (one-way) continuous functions and homeotopy is restricted to self-homeomorphisms (in the same dimension), total reflection in homeomorphic spaces cannot amount to identity in the case Voevodsky's univalent foundations and Mochizuki's inter-universal geometry with an ordinal level γ overarching transfinite dimensions can project only blurred images without homeomorphic continuity, since transfinite neighbours and their properties are refractively obnubilated (more so at *critical points* of elementary embeddings for non-self-mapping ordinals, e.g., measurable cardinals) in the reflection of the universe as can be shown in the generic sets of forcing theory and the collapsing functions of ordinal analysis with omega and aleph *fixed points*. This situation generates impredicative phenomena that cannot be reduced safely to a predicative theory, that is a non-infinitistic theory. Notice that homeomorphic embeddings in topology and geometry are generated in spaces and subspaces of the same dimension as it is the case for isotopy and that in algebra and order theory, isomorphic embeddings preserve the same cardinality. As for constructive type theory, climbing down to finite dimensions from an inaccessible cardinal has a discontinuous impact for identification of finite types up to ω with the limit

$$\lim \omega = \varepsilon_0.$$

The epsilons themselves together with the denumerable ω's form an uncountable set ω_1 with the fixed point ε^{ω}, but this fixed point cannot be sent back or filtered continuously through finite and countable ordinals which can be regarded as dimensions in a set-theoretic universe. What all this means is that an ideal fixed point – homeomorphisms need fixed points -- is beyond reach and physics and mathematics (and logic) should rest content with a constructive proximate fixed point in the finite and approximate fixed point receding *ad infinitum* in the non-finite. Of course, neither set theory in its ZF version and Peano arithmetic as infinitistic theories, nor other infinitistic theories like higher category theory or topoi theory can be shown consistent or inconsistent by the finite external means of a formal system as Hilbert had hoped, but infinitistic theories do not have the means either to prove their own consistency by Gödel's second incompleteness result on consistency. Therefore, only finitistic constructive theories (like Fermat-Kronecker arithmetic as I have called it) can afford their own internal consistency and point from without to the internal inconsistency that infinitistic theories are unable to detect from within (see [6] for more critical details).

3. A Case Study

A nice case study for a constructive version of the homeomorphism result of our general no-cloning theorem would be to explore the cosmological implications of the recent LIGO experiment for the detection of gravitational waves published in *Physical Review Letters* last February [1].

LIGO's result obtained already in September 2015 will hopefully stand after the failure of the 2014 BICEP2 result *dusted* by a Planck experiment. Although the reality of gravitational waves pertains to General Relativity, it does not priviledge a cosmological model within GR. The fact that gravitational waves cannot travel faster than the speed of light rather than the instantaneous infinite speed of Newtonian gravity relies on SR and Einstein was not so sure of their existence.

The most likely candidate to account for cosmic gravitational waves seems to be the inflation theory of Alan Guth and Andrei Linde (for eternal inflation). If inflation comes before the Big Bang (maybe after a deflation), energy is insufflated through the finite-volume singularity, let us call it the *mouth*, the burning mouth of the hot, dense nascent universe. The quantum vacuum or a cosmic electromagnetic plasma ground state (after H. Alfvén) fluctuates and breathes without end nor beginning in a continuous flow for Linde's cosmology of baby-universes, in which the mouth could be replaced by an umbilical cord attached to a mother-universe. In that scenario, the cosmic background radiation is only the *sibilance* of the vacuum. Such a scenario could be repeated indefinitely or infinitely (countably or in $\aleph_0$ universes) in the chaotic cosmic landscape imagined by Leonard Susskind. String theorists are more restrictive, they are content with 10^{500} worlds. Here string theory or M-brane theory enters the picture awaiting for supersymmetry (supergravity and superpartners) in the bubbling multiverse beyond or rather below the Higgs field where different species of stringy creatures or alien branes are encountered. Only the indiscernible particles survive in the same dimension. The chaotic generation of universes does not garantee that self-similarity is not allowed, but the transgression over different dimensions induces the evaporation of homeomorphic copies (cosmic selfies!). While many fixed points can be constructed as arbitrary sequences in a Banach space, the Lefschetz fixed-point theorem, a generalization of the Brouwer fixed-point theorem, provides some of them with homotopic properties in the *same* dimension. But if you cross over dimensions, Brouwer fixed-point theorem applies along with the no-cloning theorem in a constructive way.

At any rate, can the 5 sigma rule as a measure of confidence, that is 99.9994% probability (LIGO had 5.1), be applied anywhere in such a scenario? It could at one point at least, some 13 billions years ago at the infinite density of the instant zero of the Big Bang, a highly unphysical state of the observable universe. The mathematical fact that a Euclidean space of any dimension is contractible to a point may account for the hypothetical infinitely dense point of the Big Bang, but such a geometrical punctiform entity can hardly give birth to a physical universe. Otherwise, an inflated sphere or closed ball (hypersphere or hyperball) of any finite dimension in a Euclidean space is not contractible and is not homeomorphic to that space, while the unit sphere (all points at distance 1 from a fixed center point) in an infinite-dimensional Hilbert space is contractible and is not homeomorphic up to $\aleph_0$ spaces – an n-sphere (an n-dimensional manifold) has a boundary in an $n+1$ ball whose two cobordant manifolds M and N must have the same dimension --. A gravitational wave probe could then remove the initial *too thin* singularity and replace it with the finite volume mouth of an inflated bubble or the bouncing loop of discrete space time chunks in quantum cosmology. The detection of gravitational waves would then have the effect of discarding two infinities from the physical world, the instantaneous infinite speed of Newtonian gravity and the Big Bang singularity. Other scenarios for peering further into the multiverse, for example into parallel universes through wormholes, black holes and invisible tunnels through space-time are hypothetical constructs, but deleting the point-like singularity of the Big Bang – and dressing up black-hole « naked » singularities while lighting up dark energy (or quintessence as a scalar field) -- would be enough to open up the astrophysical world vista in the same manner that Tycho Brahe helped Kepler in his astronomical research. The LIGO team of experimenters (almost a thousand people) would have played the role of Tycho for Einstein since his idea that the distribution of matter determines the metric or the geometry of space is a distant echo of Kepler's leitmotiv relayed by Riemann, Mach and Clifford: « *Ubi materia, ibi geometria* ».

From a purely mathematical point of view, Riemann was right when he imagined discontinuous multivalued functions for his theory of surfaces or n-dimensional varieties (*Mannigfaltigkeitslehre*) and Feynman could not but invoke infinite paths in the same infinite dimension for his functional integral picture of the quantum world. But cosmic intercourse like inter-universal geometry (*à la* Mochizuki) requires multiple universes that cannot be crossed over without a loss of identity: there is no unique homeomorphic filiation or causal principle in the multiplicity of worlds where there is no dialectics of the One and the Many as in Plato's dialogue *The Parmenides.*

It is only in number theory below the $\aleph_0$ limit that one can preserve the identification of algebraic structures by Fermatian descent in finite number fields; descent or hyperdescent in the category-theoretical language of algebraic geometry needs forgetful functors to transmit abstract algebraic structures to the concrete *glue* of objects and sets in a topological or *toposical* space. Such a situation is pictured in the following commutative diagram for a pullback or fiber product:

$$\begin{array}{ccccc} & & f & & \\ & \mathbf{A} & \rightarrow & \mathbf{B} & \\ g & \downarrow & & \downarrow & h \\ & \mathbf{C} & \rightarrow & \mathbf{D} & \\ & & k & & \end{array}$$

where $h \times f = f \times g$ for arrows (morphisms) h, f, g and k; vertical arrows are forgetful functors, they forget all or part of the upper structures. For that kind of extended descent, one has to ascend beyond n and beyond ω to an inaccessible γ as noted above. The lesson here is that one must remain in the same n-dimensional universe if one wants to be faithful to its self-image; multiversal self-replication being necessarily fractional or fractal, reflection beyond any given $n+1$ cosmic (observable) horizon cannot be integral.

4. Conclusion on Foundations

To conclude, I want to insist on the constructivist standpoint challenging the realist perspective. Take for example Einstein's equations for GR:

$$R_{\mu\nu} - 1/2\, g_{\mu\nu}R - \lambda g_{\mu\nu} = -8\pi G/c^4 T_{\mu\nu},$$

where R stands for the curvature tensor, $g_{\mu\nu}$ for the metric tensor, λ for the cosmological constant, G for the Newtonian gravitational constant, and $T_{\mu\nu}$ the stress (matter) -- energy tensor. The lambda constant was introduced by Einstein for a static universe and afterwards deleted – the lambda constant survives today in the form Ω_λ of the critical density of matter in the universe and the curvature of space in the presence of matter was predicted by Clifford.

Those equations are not canonical, they have many solutions or models, from de Sitter's empty universe or anti-de Sitter universe with negative curvature to Gödel's rotating universe, and they overdetermine the empirical content of the theory. Hilbert had defined this situation in terms of the analytical apparatus and its conditions of reality « *Realitätsbedingungen* » (see [7]). John von Neumann has used these notions extensively in his 1932 classic *Mathematische Grundlagen der Quantenmechanik* and conditions of reality or realizations in the Hilbert space formalism were, for example, orthogonality for vectors, linearity and hermiticity for functional operators and the finiteness of the eigenvalue problem are constraints on the realizability of the analytical apparatus of a physical theory like Quantum Mechanics. Admittedly, those are formal constraints like the constraints imposed on solutions of Einstein's field equations – constraints on Gödel's rotating universe for instance are judged excessive --, but they can be generalised as models of a physical theory to the extent that they are variable features of a canonical analytical apparatus, as Hilbert maintained. I draw the following sketch as an illustration:

PHYSICAL THEORY

AA	ANALYTICAL APPARATUS (FORMALISM, LOGICAL AND MATHEMATICAL STRUCTURES)	**AA**
↓	MODELS	↑
EA	EXPERIMENTAL APPARATUS (EMPIRICAL DATA, MEASUREMENT INSTRUMENTS, SET-UPS AND PREPARATION PROCEDURES).	**EA**

What this scheme suggests is that interpretations are indeed necessary in physics and that they should be evaluated according to the consistency of the analytical apparatus and the coherence of the relations between the analytical apparatus and the experimental apparatus via the models generated by the physical theory. Consistency is a logical, syntactical property while coherence is a semantical attribute of the scientific representation of an integral theoretical construction of the world, as Hermann Weyl used to put it [10].

The constructivist interpretation defended here is in agreement with the Weylian standpoint and can be viewed in the case of Quantum Mechanics as a variant of the Copenhagen Interpretation with explicit constructivist logical and mathematical motives in the scope of the local observer at micro- and macroscopic scales, but in the same 2- or 3- or n-dimensional space. There the local observer appears as the open relative (local) complement of a topological space or an n-dimensional manifold homeomorphic to an n-dimensional Euclidean space. As for logic, the negation involved in the local complement corresponds to a non-

Boolean or constructive notion for which we have $\neg\neg a \neq a$ and this leads naturally to a non-classical logic internal to the given physical theory. In QM and in relativity theory, the local observer is the *coupling constant* of the relational system observed-observer; those ideas have been introduced early in my critical work on the foundations of physics from a logico-mathematical point of view (see [2], [3], and [4]). The physicist Rovelli has recently developed somewhat related ideas in his conception of a relational QM (see [8]), albeit from an oecumenical and rather uncritical realist perspective. In cosmology the local observer located anywhere is a fixed point as the central observation post of the cosmic panorama at equal distance from any point on the cosmic (hemispherical) horizon of the celestial sphere which is itself bounded by homeomorphic reflections of the local isotropic universe, as required by the cosmological principle. For the local observer, everywhere is *localized*. As explained above, what is beyond the horizon boundary lives in the same dimension since the visible is cobordant with the invisible much alike the two sides of a visible full Moon hiding its other face.

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